receivable, creditors' accounts or accounts payable, notes, material or merchandise, payrolls, equipment, investments, etc., as well as to determine the profit or loss. He listed the different kinds of accounting as: Single entry bookkeeping or memorandum records; double entry bookkeeping; auditing; statistical reports; graphic charts; business systems; cost accounting; income tax reports. He discussed the problems arising in each, dwelling particularly upon: (a) The purposes of an audit, which are, to ascertain the actual financial condition and earnings of an enterprise; to make income tax reports; to detect irregularities or errors; and (b): Cost accounting, which aids in the establishment of correct selling prices; coöperation between sales department and factory; elimination of waste and the preparation of periodical balance sheets, profit and loss, and income statements.

5. Professor Manson's paper consisted of a brief discussion of some points connected with the problem of three bodies, particularly the restricted case where the motion of a body of infinitesimal mass subject to the attraction of two finite masses which are themselves revolving in circles about their common center of mass is considered. It was shown analytically and also by a geometrical construction that if the infinitesimal body is in the plane of the orbits of the two finite masses and forms an equilateral triangle with them it will under proper initial conditions remain in equilibrium in this relative position and so maintain the equilateral configuration. The question of the stability of this solution was briefly discussed. Mention was made of the fact that the six asteroids of the so-called Trojan group seem to be oscillating with respect to these equilateral triangle points of equilibrium; two being about 60° behind Jupiter and the other four about 60° in advance. The fact of the existence of three straight line points of equilibrium; one between the two finite bodies, one between the first finite body and minus infinity and one between the second finite body and plus infinity, was also mentioned.

6. Mr. Dunham's paper contained a brief discussion of the nature of patents for inventions, and the reasons why a knowledge of mathematics beyond the rudiments of the science is a useful part of the patent lawyer's equipment, as a means of checking theories and effects, confirming results, detecting fallacies, etc., and as a means of generalizing. Illustrations having mathematical bearing were drawn from the speaker's experience.

7. The paper by Mr. Lambert and Mr. Adams will be published in the next issue of this MONTHLY.

8. Professor Cairns summarized the Bravais-Galton-Pearson method of finding the correlation coefficient and the regression equations from which, for given values of one or more variables, are found corresponding values of a dependent variable with the least (mean square) deviation of that variable. He contrasted with this the case occurring frequently, where there is no reason to regard one variable as depending on the others in the sense of cause and effect. A closer fit for a line in two variables or a plane in three variables can be obtained by imposing the condition that the mean square of the *normal* distances shall be

a minimum; like results hold for more than three variables. This is a direct generalization of the standard deviation method of comparing different distributions and yields a unique equation of relation between a variable as opposed to the usually inconsistent regression equations of the Pearson method. A short process for obtaining a first approximation for the best line or plane was given.

9. In this paper Prof. Hancock showed if $c_2=(k^2+l^2)[1-m^2-p^2]$, $c_1=mk+pl$, $b_1=k+p(ml-pk)$, $a_1=l+m(pk-ml)$, $b_2=[(ml-pk)^2-k^2][1-m^2-p^2]$, $a_2=[(pk-ml)^2-l^2][1-m^2-p^2]$, ($k$, l, m, p, arbitrary rational integers), be written in the equations: $x^2-2a_1x-a_2=0$ (a_1, a_2 rational integers), $y^2-2b_1y-b_2=0$ (b_1, b_2 rational integers), $z^2-2c_1z-c_2=0$ (c_1, c_2 rational integers), the roots of these three equations, *i.e.*, the algebraic quadratic integers ξ, η, ζ, say, satisfy the equation $\xi^2+\eta^2=\zeta^2$.

10. Prof. Moore presented a statement concerning the aims and progress of the reports of the National Committee on Mathematics Requirements. The succeeding discussion, by Mr. Boldt and Miss Gugle, dealt chiefly with the report on junior high school mathematics. In the succeeding general discussion, Prof. Weaver, speaking from experience in vocational and other schools, appealed for attention to the preparation of teachers as well as of students. Prof. Wildermuth emphasized the necessity of holding students to strict account for the work assigned; Professor Kuhn related his experiences with beginning junior high school mathematics in his own family. Further participants in the spirited discussions were Professors Beatty, Bennett, Rasor, and Dustheimer, and Supt. Collicott, and Miss Amy F. Preston of the Columbus Schools.

G. N. ARMSTRONG, *Secretary-Treasurer.*

NEW INFORMATION RESPECTING ROBERT RECORDE.

By DAVID EUGENE SMITH, Columbia University.

Our knowledge of Robert Recorde, the first mathematician of any note to publish works in the English language, is very meager. In general, recent writers have trusted to the books which he wrote and to various biographical notes that have appeared,—the former affording evidence of undoubted value, but the latter being little more than tradition.

Recorde's tombstone in the parish church at Tenby, Pembrokeshire, might be expected to be of assistance, but it turns out to be a modern one, affording no information of value. The inscription is as follows (compare the frontispiece):

IN MEMORY OF
ROBERT RECORDE,
THE EMINENT MATHEMATICIAN,
WHO WAS BORN AT TENBY, CIRCA 1510.
TO HIS GENIUS WE OWE THE EARLIEST
IMPORTANT ENGLISH TREATISES ON
ALGEBRA, ARITHMETIC, ASTRONOMY, AND GEOMETRY;
HE ALSO INVENTED THE SIGN OF

EQUALITY = NOW UNIVERSALLY ADOPTED
BY THE CIVILIZED WORLD.
ROBERT RECORDE
WAS COURT PHYSICIAN TO
KING EDWARD VI. AND QUEEN MARY.
HE DIED IN LONDON,
1558.

It is well known that Recorde was a student at Oxford, becoming a fellow of All Soul's College in 1531. In 1545 he received the degree of M.D. at Cambridge, and he is known to have taught mathematics in private classes, at both Oxford and Cambridge, prior to going to London as physician to Edward VI and Queen Mary. Roger Ascham was the Latin secretary to both the king and the queen at about that time, and it is probable that Recorde's ideas of textbook making were received in part from this great educator. Of the mathematical works which he seems to have written, only four were published.[1] These are well known and need not be mentioned in this connection.

A number of years ago the Reverend Done Bushell, rector at the Harrow School, purchased at a sale in Harrow a small portrait on an oak panel about 12 inches by 14 inches in size. The painting is apparently the work of a sixteenth century artist and is much dimmed by age. In the upper left-hand corner there is the inscription:

"Rob.t Record. M.D.
1556."

although this is so dim from age as not to show in the photographic reproduction given in the frontispiece.

As to the authenticity of the painting there can be no question. The spelling of the name is not unusual, Recorde sometimes using the final "e" and sometimes not. He was not a man of such prominence that his portrait would be painted after his death, and all the evidence goes to show that we have here an authentic painting from life, made in the year 1556, and the only portrait painting known.[2] This portrait is now reproduced by the courtesy of the present owner, W. F. Bushnell, Esq., master at the Rossall School, Fleetwood.[3]

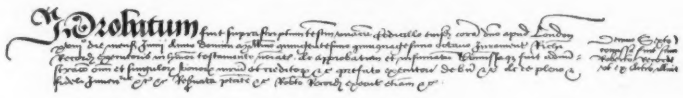
[1] The titles of these works, with the dates of the first editions, are as follows: *The Grounde of Artes*, 1541 (?); *The Castle of Knowledge*, 1551; *The Pathway of Knowledge*, 1551; *The Whetstone of Witte*, 1557.—EDITOR.

[2] W. F. Sedgwick, author of the article on Recorde in the *Dictionary of National Biography*, vol. 47, 1896, states that "the only known portraits of Recorde are woodcuts in the 'Urinal of Physick' and the 'Pathway to Knowledge.'" Consultations of copies of these works in the Library of Congress, in the Surgeon General's Library, Washington, in the Bodleian Library, Oxford, and in the Library of the British Museum, failed to reveal any such portraits. It is true that in the 1548 and 1567 editions of the *Urinal*, a cut two inches high by one and one quarter inches broad pictures the typical doctor. So also in the 1557 edition of the *Pathway* there is a figure (1 x ½ inch) of a student at his desk in the letter, G, of the word geometry. Although such cuts have not uncommonly been treated as portraits, Mr. A. W. Pollard of the department of printed books in the British Museum writes concerning the cuts here: "I do not think that there is any reason whatever to treat any of them as a portrait of Record." Hence the portrait published in connection with Professor Smith's article is probably the only existing portrait of Recorde. —EDITOR.

[3] It will also appear in the writer's forthcoming *History of Mathematics*, volume 1, together with the facsimile of the *probatum* of the will mentioned below.

It has not heretofore been published, but it was used as a basis for the bas-relief upon Recorde's monument at Tenby.

When J. O. Halliwell wrote his *Connexion of Wales with the early science of England* (London, 1840) he embodied a number of statements which, owing to his authority in matters of English History, have been generally accepted, and which were apparently relied upon in the preparation of the article on Recorde in the *Dictionary of National Biography*. Among these are the statements that Recorde's will is in the Prerogative Office and that it was dated June 28, 1558. A search for the will reveals the fact that it is only the official copy that is preserved, and that this has no date. The will was proved, as the facsimile of the official record shows, on June 18, so that Recorde must have died before that date. The *Probatum*, in reduced facsimile, is as follows:



Inasmuch as the will does not seem to have been previously printed, and is so frequently mentioned in articles upon Recorde, it is of sufficient interest and value to deserve publication. Since the official transcript is written in the legal hand of nearly four centuries ago, it is difficult to make out all of the letters, but the following copy is substantially correct:

[In the margin: T(estamentum) Roberti Recorde.] *In the name of god amen.* fforasmuch as nothing is more certaine to man then deathe, and nothing more vncertain, the houre and tyme thereof, therefore knowe yõ [= that] mi^{e} [Mr.? me?] Robert Recorde doctor of physicke though sicke in bodye yet whole in mynde thanckis be to god make my last will and testament in manner and fourme following. ffirst I comitt my soule unto thandes [= the hands] of the same allmighti god my only maker and redemer trusting by the merites of his passion to be one of his electe in glorye foreur [= forever]. my bodye as receyved from the earthe I bequeath thither again to be buryed among other christians according to the solempne vsage of the churche, my temporal goods and chattalles I ordre wille and dispoas in manner and fourme following. Secondly, I geve to Arthure hilton vnderm'shall [= undermarshall] of the kings benche whereat I now remayn prisonner xxd Item to his wife other xxd. Item to the gent [?] noix [?]prisonners w^{t} me xxxd. Item I give other xxd to the said Arthure hilton to be by him distributed amonge thofficers according to his discreation. Item to his wif to be distributed amonge her women vj s viij d. Item I give generally to the common gaole of the saide prisonne xl s to be equally distributed amonge the prisonners there. Item I give and bequeathe to myid [?] anone [?] mother and to my father in lawe her husbande xx li. Item I give to my s . . nnte (= servant) John xj li Item I give unto the children of John Battyn xl s to be distributed at the discreation of their saide father. The resideue of all my goods and chattalles moveable and unmoueable reall and psonnal. I give and bequeath

unto my brother Richard Recorde and Robert Recorde his sonne my nephewe whome I make and ordayne my full and hole executorns to thende that they beside my funeralles of the same shall truly and faithfully pay my detts. Whiche are to Nicolas ffulythm [?]citizen and merchanntaill^{o} [= merchant tailor] of London fiftie poundes to M^{r} Battyn xl s [?].

Memorand^{o} thatte saide testato^{r} on the morrowe next after the making of his testa^{t} aforesaide being then of his parfite mynde and memorye adding to the said testament gave and bequeathed to Alice and Rose Recorde daughters of the saide Richarde Recorde, and also unto Julian Raye all his vtensiles or householde stuff to be egallie diuided betwene them. Item he willed and diuised that Nicolas Adames then being prisonner in the kinges bench shulde have all his bookes concerning the lawes of this Realme at the price of iiij li. Witnes hereunto Richard Corbett George Marten and Richard Thymylby.

The tradition that Recorde was imprisoned for debt is not borne out by any evidence that I have been able to find. A search in the King's Bench Records has thus far been unproductive. There seemed to be a chance to find some evidence of payments to him as royal physician, or as Comptroller of the mines, but this, too, has not materialized, and the Memoranda Rolls and the Exchequer records are equally barren of information. In the printed Privy Council Acts, however, there are various entries relating to him as Comptroller of Mints and Monies in Ireland. There are also several entries about him in the Irish State Papers, from which entries it may be inferred that he was imprisoned owing to some misdemeanor in connection with the mines in Ireland. There is also a mention of the will in the *Index of Wills Proved in the Prerogative Court of Canterbury*, (volume 2, p. 257, London, 1898), but it contains no information of particular value. These sources are given as possible aids to anyone who may be desirous of pursuing the interesting historical investigation further. It may be added that Recorde was not Comptroller of the Mint at Bristol, as is often asserted; but the official documents show that this office was held by one Richard Recorde, presumably the brother who is referred to in the will. The statement that Recorde died "probably not long after making his will, June 28, 1558" (as in the *Dictionary of National Biography*) is also incorrect. The "XVIII" of the Probatum was probably read "XXVIII" by Halliwell or some earlier writer, and the error has been repeated by later biographers.

Editor's Bibliographical Note.

The article on Robert Recorde in the *Dictionary of National Biography*, volume 47, 1896, contains nearly forty references to various writings dealing with his life and works. The following additional references may be given:

M. Cantor, *Vorlesungen über Geschichte der Mathematik*, vol. 2, 2d ed., Leipzig, 1900, pp. 477–480, 552, 608, 621, 721, 791.

G. Eneström, "Sur l'algèbre de Robert Recorde (1546)," *Bibliotheca Mathematica*, 1901, series 3, vol. 2, p. 152.

G. Wertheim, G. Eneström, F. Amodeo, *Bibliotheca Mathematica*, series 3, vol. 3, 1902, p. 117; vol. 7, 1907, p. 290; vol. 8, 1908. p. 407.

J. Knott, "Robert Recorde, a pioneer mathematician, astronomer, and physician," *Indian Medical Record*, Calcutta, 1904, vol. 25, pp. 1–3.

D. E. Smith, *Rara Arithmetica*, Boston, 1908, pp. 213–221, 253, 286–288. [Title pages of *The Ground of Artes*, 1558, and *The Whetstone of Witte*, 1557, are here reproduced; also pages from these works showing counter reckoning and the explanation of the use of the sign for equality].

H. Zeitlinger and H. C. S., *Bibliotheca Chemico-Mathematica*, Catalogue 682 of Henry Sotheran & Co., London, 1908, p. 197; (also in *Bibliotheca Chemico-Matematica*, vol. 1, London, 1921, p. 197. There is here, opposite page 200, a facsimile page from *The Castle of Knowledge*, the first English work recognizing the Copernican system.)

L. C. Karpinski, "The Whetstone of Witte (1557)," *Bibliotheca Mathematica*, 1913, series 3, vol. 13, pp. 223–228.

J. Knott, "Robert Recorde," *Nature*, Dec. 7, 1916, vol. 98, p. 268, see also p. 172.

F. P. Barnard, *The Casting-Counter and the Counting-Board. A Chapter in the History of Numismatics and Early Arithmetic.* Oxford, 1916, pp. 254–266.

F. Cajori, *History of Elementary Mathematics with Hints on Methods of Teaching.* Revised and enlarged edition. New York, 1917, pp. 183–188; numerous other page references are given in the index.

F. V. Morley, "Finis coronat opus," *Scientific Monthly*, 1920, vol. 10, 306–308.

ON A DIOPHANTINE PROBLEM.

BY O. D. KELLOGG, Harvard University.

Professor Carmichael has been kind enough to take an interest in, and give some currency to, a problem in Diophantine analysis which I communicated to him some years ago.[1] I stated to him at that time that the maximum value of any of the unknowns that can occur in a solution in positive integers of the equation

$$\frac{1}{x_1}+\frac{1}{x_2}+\cdots+\frac{1}{x_n}=1 \tag{1}$$

was u_n, where $u_1 = 1$, and

$$u_{k+1} = u_k(u_k + 1). \tag{2}$$

His remark, in the review cited, that a complete theory of this equation seemed desirable stimulated me to attempt to reconstruct the proof of my statement, which proof I do not seem to have preserved. The attempt, however, has given rise to a doubt in my mind as to my ever having had a really valid proof, but at the same time to confirm my belief in the accuracy of the statement.

It therefore seems to me proper to make the above confession to such of the readers of the MONTHLY as may have taken an interest in the problem, and to tell what I do know about it, for it has some rather nice aspects. Diophantine problems have at least the conspicuous merit that many phases of them are quite intelligible without profound knowledge of analysis, and for that reason are frequently a stimulus to mathematical interest.

The problem arose in connection with the familiar mapping of the surface of a triangle in the z-plane upon the upper half of the w-plane by means of an analytic function of a complex variable, $w = f(z)$. If the mapping is extended by

[1] See his *Diophantine Analysis*, New York, 1915, p. 115; also his review of Dickson's *History of the Theory of Numbers*, vol. 2, in this MONTHLY, *1921*, 77.

reflections of the triangle in its sides, and thus gives rise to a single valued function, it must be possible to cover the plane with non-overlapping triangles congruent with the given one. That is, if the angles are $a\pi$, $b\pi$, $c\pi$, then a, b, c must be aliquot parts of unity with sum equal to 1. It was an extension of the question of determining what those angles could be which was formulated in the above problem.

In a certain sense a problem may be considered as solved when it is shown how its solutions may be determined by a finite number of trials. For instance, the problem of finding the rational roots of a polynomial equation with integral coefficients may be regarded as settled by the theorem that if m and n are relatively prime, m/n cannot be a root of the equation unless n is a factor of the coefficient of the highest power of the variable present in the equation, and m is a factor of the coefficient of the lowest power. In the case of the problem of finding the positive integral solutions of the equation (1), it is clear that for $n > 1$, 2 is a minimum value for any unknown. Hence all that is needed in order to show that the solutions can be found by a finite number of trials is the determination of a finite upper limit for the numbers which may occur in a solution.

I therefore offer the following "proof" of my statement about the maximum of an x_i. Regarding as identical solutions obtained from each other by a permutation of the values of the x_i, we may fix our attention on those in which the x_i form a monotone increasing sequence. Then x_n will be a maximum when $(1/x_1) + (1/x_2) + \cdots + (1/x_{n-1})$ has "exhausted" 1 to as great a degree as possible, *i.e.*, when $1 - [(1/x_1) + (1/x_2) + \cdots + (1/x_{n-1})]$ has the least possible positive value. The greatest exhaustion of 1 which is possible by one term is obtained by taking $x_1 = 2$. This leaves 1/2. The greatest exhaustion of this 1/2 which is possible by the next term is obtained by taking $x_2 = 3$. The greatest exhaustion of the remaining 1/6 which is possible by the next term is obtained by taking $x_3 = 7$, and so on. In general, if u_k is the maximum value of x_k in the equation (1) with $n = k$, we exhaust 1 best with k terms by leaving $x_1, x_2, x_3, \cdots x_{k-1}$ unchanged, giving to x_k the value $u_k + 1$. This leaves

$$\frac{1}{u_k} - \frac{1}{u_k + 1} = \frac{1}{u_k(u_k + 1)}$$

for the $(k + 1)$th term, *so that* $u_{k+1} = u_k(u_k + 1)$ *is the maximum value of an* x_i *in the equation* (1) *with* $k + 1$ *terms.*

The "proof" is phrased as if entirely valid in order to allow the reader, if so disposed, to criticize it before reading the exposé given now. The logical lapse, of course, consists in the implied assumption that the minimum of a function (here $1 - (1/x_n)$) may be obtained by minimizing it *successively* with respect to each independent variable rather than *simultaneously* with respect to them all.

A few remarks seem worth while. First, it should be observed that the method sketched always gives *a* solution, namely $x_1 = u_1 + 1$, $x_2 = u_2 + 1$, $\cdots$, $x_{n-1} = u_{n-1} + 1$, $x_n = u_n$. Further, while space is lacking to give all the

considerations which tend to substantiate the correctness of the assertion about the maximum x_n, one or two are worth mentioning. It will be observed that x_n is not greater than the least common multiple of all the other x_i, as may be seen by subtracting the sum of their reciprocals from 1, and that it is equal to this least common multiple if they are all relatively prime. It therefore appears plausible that the maximum x_n will occur when the other x_i are all relatively prime. But this is a property of the number sequence $u_k + 1$. To show this, I form the difference equation for $v_k = u_k + 1$:

$$v_{k+1} = v_k^2 - v_k + 1. \tag{3}$$

Iteration shows that v_{k+m} is a polynomial in v_k with constant term 1, so that no divisor of v_k can divide v_{k+m}. The first four values of v_k are 2, 3, 7, 43, so that one might be tempted to guess that he had here a sequence of primes. He would be promptly undeceived, however, on finding that the next number, 1807 has the factor 13. Indeed it appears difficult even to decide whether or not the sequence contains an infinite number of primes. It is, however, a simple matter to show that certain primes never occur as factor in any of the v_k, for instance 5 and 11.

That the solution containing the maximum x_n is far from the symmetric one in which all the x_i are equal ($x_i = n$), is evidenced by the following theorem. *In the solution containing the maximum x_n, no two of the x_i can be equal, except possibly the last two.* For, if $x_i = x_{i+1}$, we have the equation in $n - 1$ unknowns

$$\frac{1}{x_1} + \frac{1}{x_2} + \cdots + \frac{2}{x_i} + \frac{1}{x_{i+2}} + \cdots + \frac{1}{x_n} = 1,$$

which, on dividing by 2 and adding 1/2 to both sides, becomes the equation in n unknowns

$$\frac{1}{2} + \frac{1}{2x_1} + \frac{1}{2x_2} + \cdots + \frac{1}{x_i} + \frac{1}{2x_{i+2}} + \cdots + \frac{1}{2x_n} = 1,$$

where x_n has been replaced by a larger integer. Furthermore, in the solution containing the biggest x_n, not even the last two denominators can be equal *and even*, if $n > 2$. For $2/x_n$ can be replaced by the sum with a larger denominator

$$\frac{1}{\frac{x_n}{2} + 1} + \frac{1}{\frac{x_n}{2}\left(\frac{x_n}{2} + 1\right)}.$$

It seems as if the difference equations (2) and (3) might merit study, not only because they are probably the simplest type of non-linear difference equations, but because of some rather interesting arithmetical properties of their solutions. I will only remark that the second gives a rapidly convergent series for unity. It is

$$1 = \frac{1}{v_1} + \frac{1}{v_2} + \frac{1}{v_3} + \cdots, \qquad \text{or} \qquad 1 = \frac{1}{2} + \frac{1}{3} + \frac{1}{7} + \frac{1}{43} + \frac{1}{1807} + \cdots.$$

Not that it is recommended to use the series for computing the value of its sum approximately, but that Liouville[1] has proved the existence of infinitely many transcendental numbers by setting up rapidly convergent numerical series whose sums are transcendental. The present series does not converge as rapidly as those given by him, but it is more rapidly convergent than the exponential series for $x = 1$, which defines the transcendental number e. In fact v_n lies between $2^{(2^n-2)}$ and $2^{(2^n)}$.

AMONG MY AUTOGRAPHS.

By DAVID EUGENE SMITH, Columbia University.

10. Delambre Aids in Freeing Spencer Stanhope.

Not only was Delambre influential in securing the release of the founder of the Smithsonian Institution from prison in Hamburg, as stated in a preceding article,[2] and thus in an indirect way in the establishing of the Institution itself, but he was of service to British science and art in performing a similar service for Spencer Stanhope, a young member of the famous Stanhope family and one who at that time gave much promise in antiquarian research.

One of Delambre's letters in my collection reads as follows:

The Perpetual Secretary for mathematical sciences, to His Excellency the Minister of War.

Monsieur Le Duc,

A young Englishman, M. Spencer Stanhope, made a prisoner of war through a series of unhappy circumstances surrounding his peaceful voyage to Greece, has begged the Imperial Institute to interest itself in his favor with respect to Your Excellency.

The research which M. Stanhope is undertaking is concerned particularly with antiquities, and this is the reason which has determined the Class of Ancient Languages to recommend to Your Excellency the [favorable consideration of the] petition of this young traveler. He also includes in his plan the rendering of aid to geography, to astronomy, and to all the sciences; he is supplied with telescopic octants and he proposes in his travels to join certain geographers with whom he expects to work in Greece and the adjoining countries. His undertaking will be of service to practically all Classes and sections of the Imperial Institute. He goes out from England in search of new information which he proposes to take back for the common advantage of all civilized nations. The Imperial Institute, in applauding his designs, believes that it participates in the intentions of a generous government in recommending to Your Excellency the [granting of the] request of M. Stanhope, who earnestly desires that he may be permitted to continue the voyage so unhappily interrupted.

I am, with respect, Monsieur Le Duc,
Your Excellency's very humble and
very obedient servant,

DELAMBRE.

The letter bears an official memorandum of reference to the same official as the one relating to James Smithson, and doubtless was equally successful in accomplishing its purpose.

11. Voltaire and Mathematics.

Of those who admire that greatest champion of popular liberty in the eighteenth century, it is not probable that one in a thousand connects Voltaire's

[1] *Journal de mathématiques pures et appliquées*, series 1, vol. 16, 1851, pp. 137ff.

[2] No. 1 of this series, *1921*, 64-65.

name with the science of mathematics. Perhaps it should be so, since the candle that he placed at the altar of science was dimmed by the great lights which he set before his shrines erected to literature and to political justice. And yet there was no man living in France in his time who did so much to make the philosophy of Newton known to the literati of Paris as this same Voltaire. He knew few of the details of mathematics, but he had read and admired the *Principia*, he had caught the spirit of the work, and he wished to bring this spirit to the attention of the literary world of France. No one could have done this so successfully as he, because, while half the literati hated him, all read his every written word. He first came really to know England when Newton was dying. He visited the niece, Mrs. Conduitt; from her learned the story of Newton and the apple; and twice repeated this story in his works. He was at Newton's funeral in Westminster Abbey, and one of his biographer's remarks, "When Voltaire was very old it is said 'his eye would grow bright and his cheek flush' when he said that he had once lived in a land where 'a professor of mathematics, only because he was great in his vocation,' had been buried 'like a king who had done good to his subjects.'"

He knew Maupertuis, and despised him; knew Koenig, "a very good mathematician and a very dull man," and became his champion; knew D'Alembert, and worked with him on the *Encyclopédie*; and knew—too well—the Marquise du Chastellet and assisted her on her *Principes Mathématiques de la Philosophie Naturelle*.

His own work on Newton's philosophy (*Elémens de la Philosophie de Neuton*) contains but little mathematics. It appeared twice in the same year (1738), in Amsterdam and in London, and made Newton's mathematics known to others if not to its author.

Among my autographs are two letters written by Voltaire. One refers to his *Oreste* which appeared in 1749 and was apparently written about that time. It begins in the style that Houdon had in mind when he chiseled the well-known bust that looks at the visitor when he walks through the foyer of the Théatre Français,—

> The old invalid of Ferney has written a very consoling letter to friend Pankouke, but the true consolation consists in having many purchasers.
> The good old man is well aware that there should be a modification of the last scene of *Oreste*,—[and so on, with a closing line]
> Ut ut est, mille amitiés.

Voltaire delighted to speak of himself as "the old invalid of Ferney." He was always ill, and always at work. For half a century he was dying, and finally passed away at the age of eighty-four, with intellect clear and with pen as vitriolic as ever.

Pankouke (to take Voltaire's spelling) was his publisher. *Oreste* had not been a financial success. He had changed and changed it during its short life on the stage, until one of the actresses rebelled and declined to receive Voltaire or his letters. The story goes that he finally sent her a pâté of partridges for a dinner she was giving,—and each partridge held in its beak a note containing changes in her rôle!

The second letter was written at Ferney on April 30, 1766, and is addressed to M. le Chevalier de Taulès. Voltaire was then seventy-two years old and his

handwriting had begun to show the advancing years; but his pen had not lost its cunning nor had his sarcasm begun to fail. It is, like others of his earlier years, a plea for human rights,—this time for two men in prison in Geneva.

"I do not know the rubrics of the city of Calvin—and I do not wish to know them! Twenty citizens have come to see me, as once the fishwives of Paris paid me a similar honor. I prepared for the latter a little compliment for the King, which was well received; and I did the same for these citizens, but this has not been received in similar fashion. It appears that certain gentlemen of twenty-five are greater seigneurs than the King. I did not know that fishwives had greater privileges than such citizens; but I ask your protection for these poor devils who know only that they exist. I do not speak of the 'perruques quarrées' but of these native citizens."

This is not mathematical; it is not the letter of a mathematician; but it gives a personal view of a man who did something for mathematics and everything that man could do in the middle of the eighteenth century for human liberty.

QUESTIONS AND DISCUSSIONS.

EDITED BY W. A. HURWITZ, Cornell University, Ithaca, N. Y.

NEW QUESTION.

The discussion below by Professor Bradley leads to the request for consideration by readers of the MONTHLY of the following question:

45. Is every non-trivial solution in integers of the equation $t^3 = x^3 + y^3 + 1$ expressible in the form $x = 9r^4 - 3r$, $y = 9r^3 - 1$, $t = 9r^4$? If there are non-trivial solutions not expressible in this form, can a general solution be found?

REPLIES.

30. (*1916*, 88, 354; *1920*, 114, 362; *1921*, 124). A certain Normal University wishes to offer thirty-five hours of college mathematics for the benefit of high school teachers. What should these courses be in order that, primarily, they may be of the greatest value to high school teachers of mathematics and, secondarily, that they may furnish stimulus for a more extended pursuit of the subject?

REPLY BY T. G. RODGERS, New Mexico Normal University.

It is disappointing that the above question has not called forth a full and varied discussion from which, at least, a certain minimum of essential courses might be agreed upon. Such a discussion would be of great value to a certain class of institutions and the secondary schools with which they are closely associated.

Readers of the MONTHLY may have considered that the replies to Question 31 (*1916*, 395–399), included the above. These excellent articles gave existing conditions in standard colleges and universities but had no reference to normal schools.

Until comparatively recent years no professional training of college rank for the benefit of secondary teachers of mathematics existed in this country and the normal schools confined their efforts to the training of teachers for the grades. At present, as the above replies point out, many colleges, especially the universities with large graduate departments, offer some courses designed primarily for teachers of secondary mathematics. The normal schools which are collegiate institutions, but not graduate institutions, are gradually teaching other college subjects besides their traditional two years of professional training for teachers of the grades.

The colleges and universities, however, though offering courses in the teaching of subjects, put the emphasis upon the development of an atmosphere conducive to the pursuit of scholarship and research rather than good teaching, while the normals lay the emphasis upon the development of enthusiasm for good teaching rather than for research. The universities supply the teachers for the large high schools which can afford to pay for graduate training; the normals supply the

best grade teachers. This leaves the small high schools, many offering less than a four years' course, which abound in the nation, without provision. This class of schools has been forced to recruit its teachers too largely from the grade, teachers who have had no direct training in the subject they are called to teach other than that gained in the secondary school.

With conditions as they are, the normal schools, by extending the scope of the work which they are doing, can best serve this class of high schools and also the rapidly developing junior high schools. For these schools thorough training in the best methods of teaching a subject with a fair amount of scholarship is of much greater value than more extended scholarship and indifferent methods of teaching.

Normal schools cannot offer more than thirty-five or forty semester hours of work in any particular subject and remain distinct institutions with a definite task to perform. Otherwise they lose their first love and become mere duplications of colleges. What should these courses be in mathematics? Who can say? For the purpose of provoking a valuable discussion, the writer ventures to suggest the following:

Plane trigonometry, three hours; algebra, three; analytic geometry, five; differential and integral calculus, six; modern geometry, five; the pedagogy of arithmetic, two; the pedagogy of algebra, two; the pedagogy of plane geometry, two; the pedagogy of solid geometry, two; observation in the practice high school, two; the remaining three hours to be given to a further study of analytic geometry, calculus or modern geometry, as the particular class elects. For the ends sought these introductory courses are better than longer courses in fewer subjects.

The work in trigonometry should emphasize the use of tables, the slide rule, the solution of triangles and a fair amount of theoretical work including inverse functions. The algebra should put the emphasis upon theory and logic rather than technique, and center around the simple progressions, quadratics, combinatory analysis, the theory of equations and the determinant. In analytics the straight line, the circle, functions and graphs, and the transformation of co-ordinates should receive the emphasis. For the purpose in view a six hour course based on some recent text in calculus of the type of that of March and Wolff will yield richer results than the more formal work in differential and integral calculus of the older type.

A high school teacher who can give but five hours to modern geometry will gain more that is of direct value to him if he divides it into a two hour course in simple theorems pertaining to concurrent lines, collinear points, the homothetic transformation, the transformation by inversion, the radical axis, coaxal circles and possibly polar reciprocal figures and applies them to construction work. This should be followed by a three hour course in synthetic projective geometry.

In each of the pedagogical courses a portion of the time should be given to a serious study of the history and literature of the subject of the course. There is an immense amount of material for this phase of the work. It must be gained from elementary histories of mathematics; books on the teaching of mathematics of such authors as Smith, Young and Stone; government bulletins on the teaching of mathematics, including the reports of the National Committee on Mathematical Requirements and the International Commission on the Teaching of Mathematics; and text books of different types, including some of foreign countries.

In addition to this purely pedagogical phase the prospective teacher needs the special study of the most difficult parts of each subject. In algebra he should take a dip into analysis, studying such topics as the number system, irrational numbers, limits, infinite series and topics that the individual class seems to need. In plane geometry he should get a survey of the foundations of geometry, become acquainted with the famous problems of antiquity and as far as time permits study the subject from the larger view of such works as Hadamard, *Leçons de Géométrie*. In solid geometry this larger view point so essential to a teacher can be gained by the study of selected portions of such works as Hadamard, *Leçons de Géométrie dans l'Espace*.

It is highly desirable that secondary teachers of mathematics become acquainted with the scholarly ideals of some of the best foreign works on elementary algebra and geometry. But for such a brief course the professor will have to make the selections and give them in lecture form to be worked up by the students.

With this as a foundation the students are in fine condition to observe in the practice high school, and meet one hour a week for conference and discussion of what they have observed. For a semester's work of this kind two hours of college credit seem sufficient.

Since this work is taken in a normal school it is understood that these students have also had the regular training in psychology and education.

It is admitted that this is a brief course in mathematics for a secondary teacher of mathematics. But until the standards of preparation for this class of teachers can be raised throughout

the nation, the writer believes this represents a step in the right direction for the preparation of teachers for the junior high school and the small high school.

DISCUSSIONS.

The two short discussions seem to require no comment. We should be glad if Question 45, printed above, which is called forth by Professor Bradley's discussion of the Diophantine equation $t^3 = x^3 + y^3 + 1$, should receive some interesting replies. Mr. Roman's note on a humble phase of the mathematics of investment should be entertaining.

I. On a Diophantine Equation.[1]

By H. C. Bradley, Massachusetts Institute of Technology.

On page 77 of the February number of the Monthly, Professor R. D. Carmichael asks, among other things, for a general solution of the Diophantine equation $t^3 = x^3 + y^3 + 1$.

I wrote to Professor Carmichael, calling attention to the following. In his own *Diophantine Analysis*, p. 65, he gives a general solution of $x^3 + y^3 = u^3 + v^3$ as follows:

$$x = -(a - 3b)(a^2 + 3b^2) + 1, \qquad y = (a + 3b)(a^2 + 3b^2) - 1,$$
$$u = -(a^2 + 3b^2)^2 + (a + 3b), \qquad v = (a^2 + 3b^2)^2 - (a - 3b).$$

Now let $a = 3b$, then let $2b = r$, and change the variables, and we have

$$x = 9r^4 - 3r, \qquad y = 9r^3 - 1, \qquad t = 9r^4;$$

which is a solution of $t^3 = x^3 + y^3 + 1$.

Or, if we assume the possibility of a solution of the form $x = Ar^4 - Br$, $y = Cr^3 - 1$, $t = Ar^4 + Dr$, the coefficients may be determined as above.

This solution gives $9^3 = 6^3 + 8^3 + 1$, $144^3 = 138^3 + 71^3 + 1$, $729^3 = 720^3 + 242^3 + 1$, etc. It fails to include the trivial case $x = y = 0$, $t = 1$.

Professor Carmichael replied that this was interesting, but asked if I could prove that it gave all non-trivial integral solutions of the given equation. This I cannot do. So at his suggestion I am sending the result, thinking that perhaps some other readers of the Monthly might be interested to work on it.

II. A Note on War Savings Stamps.

By Irwin Roman, Northwestern University.

The accompanying table illustrates an interesting property of the War Savings Stamps issued by the government in 1918. A similar table holds for later issues. The third column gives the amount which the post-office will pay for each five dollar stamp during the month. If we consider this amount as reinvested, the fourth column gives the profit accrued at maturity, while the fifth column gives the per cent. this profit is of the amount thus reinvested. The final column shows the annual rate of interest corresponding to the case.

[1] Extract from a letter to the Editor.

Table.

Year	Month	Redemption Value	Profit Actual	Profit Per Cent.	Rate Per Year
1918	Jan.	4.12	0.88	21.3	4.3
1919	Jan.	4.24	0.76	17.9	4.5
1920	Jan.	4.36	0.64	14.6	4.9
1921	Jan.	4.48	0.52	11.6	5.8
1921	April	4.51	0.49	10.9	6.2
1921	July	4.54	0.46	10.1	6.7
1921	Oct.	4.57	0.43	9.4	7.5
1922	Jan.	4.60	0.40	8.7	8.7
1922	April	4.63	0.37	8.0	10.7
1922	July	4.66	0.34	7.2	14.4
1922	Oct.	4.69	0.31	6.6	26.4
1922	Nov.	4.70	0.30	6.4	38.4
1922	Dec.	4.71	0.29	6.2	74.4
1923	Jan.	5.00	Maturity		

While the table speaks for itself, four conclusions may be mentioned:

1. Stamps bought at the redemption value bear a rate of interest, calculated as simple interest, varying from 4.3 to 74.4 per cent. per year.

2. It becomes increasingly desirable to hold the stamps till maturity.

3. When sale is necessary, the post-office becomes an increasingly cheaper buyer than a private individual.

4. A loan with the stamps as collateral becomes increasingly desirable as opposed to sale.

For the sake of simplicity, all calculations are referred to the first day of the month and all interest is assumed to be simple. The writer has never had this aspect of the War Savings Stamps called to his attention, and believes it might be of interest, if not of value, to readers of the MONTHLY.

RECENT PUBLICATIONS.

BOOK REVIEWS.

Les Principes de L'Analyse Mathématique. Exposé Historique et Critique. By PIERRE BOUTROUX. 2 volumes, Paris, A. Hermann & Fils, 1914–1919. Royal 8vo. xi + 547 + 512 pages. Price 14 + 20 francs.

In these two volumes we have one of the most suggestive historical and critical surveys of the whole field of elementary mathematics that has appeared in recent times. The purpose of the work may be best stated in the words of the author: "Except for these restrictions [demonstrations of certain theorems and detailed explanations of technique] the present work contains all or nearly all the material contained in the course in 'general mathematics' given in our

Faculties of Science. It moreover goes beyond them notably in that it touches in some places on certain of the highest chapters of modern analysis, and on the other hand it takes up mathematical science at its origin, in its principles, in order to present as nearly as can be done a view of the whole. . . . Its first object is none the less to furnish objective information and to serve as a guide to beginners in mathematics."

If the word "beginners" is properly interpreted—let us say, beginners in the work of our graduate schools—this object has been very successfully attained. The greatest value of the work, however, is for the teacher or advanced student who wishes to gain a comprehensive view of the whole field of mathematics, either as a body of knowledge, or more especially as a mode of thought. And many parts of it will be found valuable for supplementary reading on the part of undergraduates who are interested in getting at the bottom of things.

The care with which the fundamentals are discussed may be inferred from the fact that the first chapter, on (rational) "numbers," covers 61 pages, and the second, on "magnitudes" (including irrational numbers defined both arithmetically and geometrically) covers 118 pages. After passing more rapidly over some of the important topics and methods of elementary geometry, plane trigonometry, and algebra, we reach the study of functions, to which the remainder (239 pages) of the first volume is devoted, the point of view being almost purely arithmetical. The second volume enters upon the discussion of analytic geometry in two and in three dimensions, to which about 125 pages are given; and then follows a chapter (97 pages) on "extension of algebra, and logical constructions," which includes the elementary parts of the algebra of complex numbers, the elementary parts of group theory, with an indication of their applications to the theory of algebraic and differential equations; and a discussion of the foundations of algebra and geometry. Then follow developments in series (36 pages) and infinitesimal calculus (85 pages). The last two chapters, entitled "Analysis of Mathematical Principles" and "Analysis of the Notion of Function" cover 37 and 71 pages respectively. The last chapter includes the elementary properties of functions of a complex variable, such as Cauchy's integral theorems, Taylor's series, singular points, analytic extension, and simply and doubly periodic functions, with mention of elliptic functions and integrals.

As an example of the methods of treatment which are employed, we may take the subject of the conic sections. They are first taken up as sections of a cone, in vol. I, pages 241ff., where the work of the Greeks is cited, without however being described in detail; then the curves are discussed more at length as plane loci, but without any use of coördinates; and only in vol. II, pages 50–65, do we meet with the study of these curves from their equations. At this point it seems regrettable that the author has let slip the opportunity to point out the connection between the locus problem and the equation; there is no intimation that the equation could have been obtained from the definition of the curve as a locus, but only the converse procedure is mentioned, and that, too, starting from the general equation. Another detail of arrangement seems to the re-

viewer defective: that determinants are taken up immediately *after* the study of transformations of coördinates has been completed; had this order been changed, the classification of equations of the second degree could have been stated in a neater and more concise manner, and much of the work in analytic geometry of 3 dimensions could likewise have profited. Indeed no mention is made in connection with determinants that they are used at all in geometry.

The author's view of the nature of mathematical analysis is brought out clearly in the admirable introductory chapter to the third book (volume II, pages 259–267). Analysis, as understood today, may be regarded as a combination and reconciliation of the two tendencies which have been more or less in opposition one to the other throughout the whole history of mathematics: the tendency to look outward upon the world and seek to understand and explain it, and the tendency to look inward to the forms of thought itself, and to develop methods of reasoning for their own sake. Modern analysis is in fact concerned with both these points of view, and Professor Boutroux has summarized the historical developments in such a way as to make the fact stand out strikingly. The same subject is continued and amplified in the penultimate chapter, where the fundamental relations of number and magnitude are subjected to a careful scrutiny, in the light of the results of the infinitesimal calculus. It is brought out clearly that in the modern idea of the irrational number we have the bridge that connects the discrete world of (cardinal) *numbers* with the continuous world of (geometric) *magnitude*. Thus we gain a firm basis for the concept of continuity, and the general notions of the function theory, with which the book concludes.

A few slips occur: thus, vol. I, p. 20, line 4 from the end, *n times unity* should be $(n + 1)$ *times unity*, and similar changes should be made three times at the top of p. 21. On p. 38, at the end of § 31, the statement that "the first rigorous and complete exposition of calculation with fractions is found in Stevin's *Arithmétique*" is too sweeping, as (to mention only one name) Leonardo of Pisa[1] gives an account that leaves nothing to be desired—unless probably brevity. On p. 139, the two expressions are equal $(= 4/\pi)$ instead of one being $= 2/\pi$ as stated. On p. 183, Thales is said to have had scarcely any but a practical knowledge of geometry; this seems overdrawn in view of Proclus's ascription to him of "general treatment" (*καθολικώτερον*) of certain problems, along with the general, if less well-authenticated, crediting of the theorem of the right angle inscribed in the semicircle to him. On page 521, Fermat is credited with having stated "very clearly the conception of the derivative and the characteristic [differential] triangle." As to the derivative, this is surely overdrawn, and it seems doubtful if it can be maintained as to the differential triangle. In vol. II, page 58, the word "supplementary" in the next to the last line should be replaced by "one exceeding the other by π."

There is an unfortunately large number of misprints, even after those noted in the "Errata" have been corrected; perhaps those most likely to be confusing are vol. I, page 90, next to last line, read *parallèles* for *perpendiculaires*; page 146,

[1] *Scritti di Leonardo Pisano*, Rome, 1857, vol. I, pp. 47–83.

line 4 from end, interchange *grande* and *petite*; page 169, lines 5 and 6, interchange *sinus* and *cosinus*; page 183, line 4 of the note, for *qualité* read *quantité;* page 266, line 11, for *aa, bb, cc*, read *a, b, c*; page 406, last and 4th from last lines, interchange *maximum* and *minimum;* and similarly on page 522, where in the last line the inequality signs should be reversed; vol. II, page 56, the figure should be numbered 245.

None of these minor blemishes, however, can affect the final judgment of the work, which is that through its clear presentation and discriminating evaluation of so many of the most important topics in the whole field of mathematics, as well as through its stimulation to further thought and investigation along all lines, this book is a really great contribution to mathematical literature, which should be in the possession of every teacher.

R. B. McClenon.

Tables of the Exponential Function and of the Circular Sine and Cosine to Radian Argument. By C. E. Van Orstrand. (*Memoirs of the National Academy of Sciences*, volume 14, fifth memoir.) Washington, D. C., 1921. 4to. 79 pp.

Mr. Van Orstrand, a physical geologist of the U. S. Geological Survey, and a charter member of the Association, has once more rendered fine service to workers in certain fields of applied mathematics. In 1909 he collaborated with G. F. Becker in publishing the 360 page volume, *Hyperbolic Functions* (Smithsonian Mathematical Tables). He has now published fourteen tables concerning which some preliminary publications appeared in *Journal of the Washington Academy of Sciences*, 1912–13. The tables are as follows:

I: Values of the reciprocal of $n!$ to 108 places of decimals at intervals of unity from 1 to 74; II: Values of e^x to 42 significant figures at intervals of unity from 0 to 100; III: Values of e^x to 33 significant figures at intervals of 0.1 from 0.0 to 50.0; IV: Values of e^x to 62 places of decimals at decimal intervals from 1×10^{-14} to 9×10^{-1}; V: Values of e^{-x} ranging from 52 to 62 places of decimals at intervals of unity from 0 to 100;. VI: Values of e^{-x} ranging from 33 to 48 places of decimals at intervals of 0.1 from 0.0 to 50.0; VII: Values of e^{-x} to 62 places of decimals at decimal intervals from 1×10^{-10} to 9×10^{-1}; VIII: Values of $e^{\pm (n\pi/360)}$ to 23 places of decimals or significant figures at intervals of unity from $n = 0$ to $n = 360$; IX: Values of $e^{\pm n\pi}$ to 25 places of decimals or significant figures for various values of n; X: Values of $\sin x$ and $\cos x$ to 23 places of decimals at intervals of unity from 0 to 100; XI: Values of $\sin x$ and $\cos x$ to 23 places of decimals at intervals of 0.1 from 0.0 to 10.0; XII: Values of $\sin x$ and $\cos x$ to 23 places of decimals at intervals of 0.001 from 0.000 to 1.600; XIII: Values of $\sin x$ and $\cos x$ to 25 places of decimals at decimal intervals from 1×10^{-10} to 9×10^{-4}; XIV: Miscellaneous values of e^x, e^{-x}, $\sin x$ and $\cos x$ to a great number of decimals, including Boorman's value of e.

In preparing such tables the author sought not only to obtain "a few high place values at sufficiently small intervals of argument for general use in the evaluation of integrals and other functions," but also to obtain "a basis for

subsequent interpolation to small intervals of argument for use in the construction of complete 10-place tables which are applicable in the various fields of pure and applied mathematics."

The author remarks that the most important tables of extended values of the exponential function in which the exponents are integers or fractions were those constructed by Schulze (1778), Bretschneider (1843), Newman (1883, 1889), Gram (1884), Glaisher (1883), and Burgess (1900). The extent of the contribution of each is indicated. There is no reference to Salomon's *Tafeln*, 1827, where the values of e^n, $e^{.n}$, $e^{.0n}$, $\cdots$ $e^{.000000n}$, for $n = 1, 2, 3, \cdots 9$, may be found.

The only previous table of the reciprocal of $n!$ seems to have been the one by Glaisher,[1] in 1877, to twenty-eight figures as far as $n = 50$. Mr. Van Orstrand shows that Galisher's table is in error, by one unit, in the twenty-eighth figure of each of the numbers $n = 20, 27, 41, 50$. An elaborate table of $\log_{10} n!$ from $n = 1$ to $n = 1200$ to eighteen places[2] was given in C. F. Degen, *Tabularum Enneas*, Copenhagen, 1824.

In Table IX the values of $e^{n\pi}$ are given for the following sixty values of n: $\pm 7/6$, $\pm 13/6$, $\pm 19/6$, $\pm 5/4$, $\pm 9/4$, $\pm 13/4$, $\pm 4/3$, $\pm 7/3$, $\pm 10/3$, $\pm 3/2$, $\pm 5/2$, $\pm 7/2$, $\pm 5/3$, $\pm 8/3$, $\pm 11/3$, $\pm 7/4$, $\pm 11/4$, $\pm 15/4$, $\pm 11/6$, $\pm 17/6$, $\pm 23/6$, ± 2, ± 3, ± 4, ± 5, ± 6, ± 7, ± 8, ± 9, ± 10. Three of these, namely for n equal to $-9/4$, -4, and -9, are included in the seventeen values of $e^{n\pi}$ (or $2e^{n\pi}$) given in this MONTHLY, *1921*, 115–120. On page 11 of his introduction Mr. Van Orstrand remarks that "The value of $e^{\pi/2}$ given by Gauss is incorrect in the twenty-third and following decimals"; this error seems to have been first pointed out in this MONTHLY, *1921*, 120.

In table XIV is included (except for one error[3]) the value of e to 346 places of decimals published by J. M. Boorman, "consultative mechanician and attorney at law, Brooklyn, N. Y.," in *Mathematical Magazine*, Washington, August, 1887; it is here pointed out that Shanks's computation of e (1854) to 205 places was incorrect beginning with the 188th decimal. Mr. Van Orstrand states that Tichánek and Minks verified (1892) Boorman's value of e to 223 decimal places, giving as authority *Jahrbuch über die Fortschritte der Mathematik*, volume 23, p. 441 and volume 25, p. 736. A comparison of the numbers shows that they differ in the forty-third decimal place; at that place Boorman gives (correctly) "0" and not "6."

R. C. ARCHIBALD.

April 29, 1921.

The Copernicus of Antiquity (Aristarchus of Samos). By T. L. HEATH. (Pioneers of Progress, Men of Science.) London, Society for Promoting Christian Knowledge, 1920. 4 + 59 pages. Price 2 shillings.

[1] *Cambridge Philosophical Society Transactions*, vol. 13, pp. 246–247.

[2] De Morgan gave a six-place abridgment in his article on "Theory of Probabilities" in *Encyclopedia Metropolitana*, 1837.

[3] Mr. Van Orstrand gives "0" instead of "6" in the thirty-second decimal place. This error may be verified by the computations of Shanks (*l.c.*) and of Glaisher (*l.c.*).

First three paragraphs: "The title-page of this book necessarily bears the name of one man; but the reader will find in its pages the story, or part of the story, of many other Pioneers of Progress. The crowning achievement of anticipating the hypothesis of Copernicus belongs to Aristarchus of Samos alone; but to see it in its proper setting it is necessary to have followed in the footsteps of the earlier pioneers who, by one bold speculation after another, brought the solution of the problem nearer, though no one before Aristarchus actually hit upon the truth. This is why the writer has thought it useful to prefix to his account of Aristarchus a short sketch of the history of the development of astronomy in Greece down to Aristarchus's time, which is indeed the most fascinating portion of the story of Greek astronomy.

"The extraordinary advance in astronomy made by the Greeks in a period of little more than three centuries is a worthy parallel to the rapid development, in their hands, of pure geometry, which, created by them as a theoretical science about the same time, had by the time of Aristarchus covered the ground of the Elements (including solid geometry and the geometry of the sphere), had established the main properties of the three conic sections, had solved problems which were beyond the geometry of the straight line and circle, and finally, before the end of the third century B.C., had been carried to its highest perfection by the genius of Archimedes, who measured the areas of curves and the surfaces and volumes of curved surfaces by geometrical methods practically anticipating the integral calculus.

"To understand how all this was possible we have to remember that the Greeks, pre-eminently among all the nations of the world, possessed just those gifts which are essential to the initiation and development of philosophy and science. They had in the first place a remarkable power of accurate observation; and to this were added clearness of intellect to see things as they are, a passionate love of knowledge for its own sake, and a genius for speculation which stands unrivalled to this day. Nothing that is perceptible to the senses seems to have escaped them; and when the apparent facts had been accurately ascertained, they wanted to know the *why* and the *wherefore*, never resting satisfied until they had given a rational explanation, or what seemed to them to be such, of the phenomena observed. Observation or experiment and theory went hand in hand. So it was that they developed such subjects as medicine and astronomy. In astronomy their guiding principle was, in their own expressive words, to 'save the phenomena.' This meant that, as more and more facts became known, their theories were continually revised to fit them."

Contents—Part I, *Greek Astronomy to Aristarchus*, 1–37: Thales; Anaximander; Anaximenes; Pythagoras; Parmenides; Anaxagoras; Empedocles; The Pythagoreans; Œnopides of Chios; Plato; Eudoxus, Callippus, Aristotle; Heraclides of Pontus. Part II, *Aristarchus of Samos*, 38–56: The heliocentric hypothesis; On the apparent diameter of the sun; On the sizes and distances of the sun and moon; On the year and 'great year'; Later improvements on Aristarchus's figures. Bibliography, 57–58. Chronology, 59.

Problems and Solutions. Associateship Examinations, Parts I and II, 1915–1919. New York, Actuarial Society of America, 1921. 8vo. 133 pages + 46 figures. Price $2.00.

Foreword: "The problems herein set forth with their solutions comprise all of the problems set in the years 1915 to 1919 inclusive, in Parts I and II of the examinations for admission to Associateship in the Actuarial Society of America. These problems and solutions are published primarily for the use of students preparing for these particular examinations; but they will unquestionably be of value to many who are teaching or studying mathematics in high school or college.

"Prior to 1920 these two examinations comprised what was known as *Section A* of the Associateship examination. The nomenclature has been changed so that they are now known simply as Part I and Part II of the Associateship examination.

"In many instances the solutions as set forth are not the only solutions of the given questions and it is not our intention to infer that the published solutions would have been more acceptable than any others to the Examination Committee of the Society. We have simply attempted to present a correct solution to each problem. The major part of the work of editing and arranging the solutions for publication was done by Dr. LESTER R. FORD of The Rice Institute, working in coöperation with the Educational Committee [J. M. Laird, J. F. Little, E. W. Marshall, H. N. Stephenson, and M. A. Linton, chairman] of the Actuarial Society, under whose supervision the book has been published. Charles M. Taylor, a student of the Society, rendered valuable service in reading the proof and making helpful suggestions."

The volume contains 279 problems in all: 9 in arithmetic, 51 in elementary algebra, 16 in plane geometry, 36 in plane trigonometry, 15 in analytical geometry, 9 in bookkeeping, 45 in advanced algebra, 20 in the theory of probabilities, 28 in calculus of finite differences, and 50 in calculus.

NOTES.

The concluding number of *Proceedings of the Royal Society*, London, series A, volume 98, published March 24, 1921, contains a fifty page notice of "John William Strutt, Baron Rayleigh, 1842–1919." There is also a fine frontispiece portrait.

The Mannheim & Polyphase Slide Rules. A Self Teaching Manual with tables of settings, equivalents and gauge points is the title of an 80-page pamphlet mainly by W. E. BRECKENRIDGE, associate in mathematics at Columbia University (New York, Keuffel & Esser, 1920). The supplement, "The slide rule in trigonometry" (pages 63–77), was written by Professor J. M. WILLARD, of the State College of Pennsylvania.

A. E. H. Love's *Theoretical Mechanics: an introductory treatise on the principles of dynamics, with applications and numerous examples* was first published by the Cambridge University Press in 1897. A second edition with few changes appeared ten years later. Of this, an authorized German translation by R. Polster was published in 1920 (Berlin, Springer, 14 + 424 pages. Price 48 marks). For English measures German have been substituted, and an alphabetic subject index has been added.

Various reviewers of Sir Thomas Heath's *Euclid in Greek, Book I, with Introduction and Notes* (Cambridge, 1920) [see, for example, *1920*, 263–266] seem to have overlooked the fact that only four years previously G. C. Sansoni of Florence published the very neat little volume edited by Giovanni Vacca with the following title: *Euclide. Il primo Libro degli Elementi, testo Greco, versione Italiana, Introduzione e note* (1916. 12mo. 20 + 122 pp.). On the last four pages there is a glossary of Greek words with the Italian equivalents.

The concluding number of the *Bulletin of the American Mathematical Society*, volume 27, was for June–July, 1921. It has been decided that in the future the volumes shall begin in January. Hence the first number of volume 28 is to be that for January, 1922.

Between 1913 and 1919 the following volumes of the *Opera Omnia* of Tycho Brahe have been published at Copenhagen (*1920*, 421): I, II, III, IV part 1, and VI. The first part of volume V (*Astronomiae Instauratae Mechanica*, 1598; 213 pages) has recently been published.

In *Il Bollettino di Matematica*, volume 17, 1920, pages 61–77, A. Natucci reviews for the tercentenary of the invention of analytic geometry (*1921*, 179) the edition of the works of Descartes edited by C. Adam and P. Tannery.

The second heft of *Jahrbuch über die Fortschritte der Mathematik*, volume 45, for 1914–1915, was published in July, 1921, nearly two years after the first part was issued (*1920*, 268). The concluding part is promised by November, 1921. The present heft (pages 369–944) contains, apart from the conclusion of combinatory analysis and calculus of probability, series, differential and integral calculus, theory of functions, pure elementary and synthetic geometry, analytic geometry, and three pages on mechanics. There are in the heft about 116 pages more than in the corresponding section of the *Jahrbuch* for the year 1912.

In *Revista Matemática Hispano-Americana*, January–April, 1921, there are a portrait, biographical sketch, and bibliography of the published papers of T. Levi-Civita (pages 1–10, 46–49). The list of published papers contains over one hundred titles. Supplementary to the first five numbers of the *Revista* for 1921 have been published 80 pages of a Spanish translation of *Questioni riguardanti le Matematiche elementari*, edited by Federigo Enriques, volume 1, which is possibly more familiar to American readers in the German translation. The above mentioned 80 pages cover about the same ground as the corresponding number of pages in the German translation.

We are indeed happy to learn that a considerable increase in the number of subscribers to *Journal de Mathématiques Pures et Appliquées* has made the period of suspension of publication a very brief one (compare *1921*, 134). Not only has volume 85, 1920, been published, but also (in May and August, 1921) the the first and second numbers of volume 86. The latter include an address, by Camille Jordan, pages 1–2, at the funeral of Georges Humbert (*1921*, 237), and a paper "Transformations of surfaces applicable to a quadric" by L. P. Eisenhart, 37–66.—The first four numbers of *Annales Scientifiques de l'Ecole Normale Supérieure*, volume 56, and fascicules I–II of *Bulletin de la Société Mathématique de France*, volume 49, have also appeared in 1921.

Two new Italian mathematical periodicals have been started this year by the Circolo Matematico di Catania (each 25 lire a year and printed by V. Giannotta, Catania). Of the one, *Esercitazioni Matematiche*, for the use of university students, Professor Michele Cipolla is the editor. The first number (52 pages) was for January–February. Professor Gaetano Scorza is the editor of the other, *Note e Memorie di Matematica*, fasc. 1 (64 pages).

Revista Matematică din Timişoara is the title of a periodical published by the Scoala Politecnică, for mechanics and mining, recently established (1920) in Timişoara, lately of Hungary but now of Roumania. The first number appeared in March, and the second in April, 1921. These issues contain brief articles, problems proposed and solved, notes and news, examination questions. The periodical is designed to assist those preparing to enter the Scoala.—*Gazeta Matematică*, the other Roumanian mathematical periodical, has been published

for twenty-six years and is the organ of the Society "Gazeta Matematică" with about fifty members desiring to promote the interests of mathematics in the secondary and higher schools of the country.

Sitzungsberichte der Preussischen Akademie der Wissenschaften, 1921, no. V, contains the Festvortrag, "Geometrie und Erfahrung" (pages 123–130), delivered by Einstein January 27, 1921, and an account (pages 116–123, 138–139) of the new edition of the complete works of Leibnitz. It was planned that this edition should occupy about 60 quarto volumes. By using smaller type and cutting down the editorial remarks, the commission of the Akademie now plans to reduce this estimate to 39 octavo volumes. The collected papers will be published in four series: philosophical (6 volumes), mathematical, in the field of natural science, and politico-historical; the correspondence in three series is to be classified: philosophical (6 volumes), in the fields of mathematics and natural science, and politico-historical and general (10 volumes). The 22 volumes of philosophical writings, and correspondence dealing with philosophy, politico-historical matters, and generalities are to be prepared first. One of these volumes is ready for publication.

The University of Illinois published, "November 22, 1920," an eight page illustrated pamphlet, whose contents are dated "January, 1921," entitled *Mathematical Models*. The author is Professor ARNOLD EMCH. He describes 18 models "designed and constructed in the mathematical department of the University of Illinois, . . . the first results of an effort to represent certain desirable features of mathematical instruction and research by adequate models, mechanisms, or graphs when they are not available otherwise. . . . For those interested . . . it will be possible to make arrangements with private firms for the reproduction and sale of the . . . models at prices which will be quoted upon application." The models include: (*a*) a variable string model of the hyperbolic paraboloid; (*b*) string models of certain cubic, quartic, quintic and sextic ruled surfaces; (*c*) tangent surfaces of a rectilinear (2, 2)-congruence; (*d*) models of projective generation of surfaces; (*e*) plane sections of a torus; (*f*) cellular division of space; and (*g*) three linkages—(α) Hebbert's cardioidograph, described in this MONTHLY, *1915*, 12–13; (β) a mechanism illustrating the description of certain ruled surfaces (Hebbert); and (γ) a cinematographic film of a Poncelet "polygon," *i.e.*, "a triangle remaining inscribed and circumscribed to two fixed circles respectively."

Since our last report (*1921*, 177), six more parts have been issued in *Publications of the Massachusetts Institute of Technology* "Contributions from the department of mathematics," series (since no. 7 changed from "serial") 2. These parts are number 17–22, February–July, 1921, and contain articles, by F. L. Hitchcock, Joseph Lipka, C. L. E. Moore, L. H. Rice, and Norbert Wiener (2), (see *1921*, 177, 178), reprinted from *American Journal of Mathematics, Bulletin of the American Mathematical Society, Comptes Rendus du Congrès International*

des Mathématiques, and *Proceedings of the American Academy of Arts and Sciences*. The first article in the *Proceedings*, volume 56, was "Motion on a surface for any positional field of force" by J. Lipka, in no. 4, March, 1921, pp. 157–182; the second article, "The axes of a quadratic vector" by F. L. Hitchcock, appeared in no. 9, June, 1921, pp. 331–351. The articles by N. Weiner were respectively three and four pages in length, being the papers he read at the so-called international congress of mathematicians (*1920*, 440).

Attention is directed to the interesting and admirably edited first three numbers (January–March, 1921, pp. 1–232) of *Periodico di Matematiche* (*Storia, Didattica, Filosofia*), organ of the Società Italiana di Matematiche "Mathesis," edited by F. Enriques and G. Lazzeri. Although these numbers[1] are marked series IV, vol. 1, nos. 1–3, the periodical is really an entirely new one having very little in common with its immediate predecessors *Periodico di Matematiche* and *Bollettino della Società Italiana di Matematiche "Mathesis."* The second series of the *Periodico*[2], 1899–1903, edited by A. Lugli, was formerly the official organ[3] of the Society of "Mathesis" which was founded[4] in 1895 by a group of teachers, in secondary schools, interested in the discussion of didactic questions.

For several years prior to 1908 interest in this society was decidedly on the wane, but in that year it was reorganized, in collaboration with university professors, under the name Società Italiana di Matematiche "Mathesis."

We are indebted to Professor Enriques, of the University of Bologna, for many of the facts contained in this note.

Of the ten mathematical periodicals started since January, 1919, none are of such notable importance for mathematical research as *Fundamenta Mathematicae* of which two volumes have been published; the first (224 pages) in 1920, the second (287 pages) in 1921, before May 1. The periodical is confined to the publication of memoirs, notes, and problems dealing with the theory of aggregates and related questions (immediate applications of the theory of aggregates,

[1] Among the articles in them are: "The teaching of dynamics" by F. Enriques; "The theory of irrational numbers in antiquity" by T. Bonnesen; "Paradoxes of infinity" by G. Vivanti; "Reform in the teaching of mathematics in the United States of America" by D. E. Smith; and "On the construction of a triangle given the lengths of its angular bisectors" by O. Chisne—a problem discussed by R. P. Baker in his doctor's dissertation at the University of Chicago, published in 1911 (100 pages, 4to).

[2] *Periodico di Matematica per l'insegnament secondario* was founded by David Besso in 1886. The first series contained 13 volumes, published 1886–1898. The third series, edited by G. Lazzeri, was published at Leghorn (Livorno), 1904–1920. The first volume of the fourth series is being published by Zanichelli at Bologna (20 francs).

[3] The first official organ of the society was *Bollettino della Mathesis* of which three volumes were published at Rome and Turin, 1896–1898. Under the title *Bollettino della Società Italiana di Matematiche "Mathesis"* it was published with Severi as editor 1909–1910; with Castelnuovo as editor 1911–1914; with Berzolari as editor 1915–1918; with Enriques as editor 1919–1920. It was only since about 1910 that the *Bollettino* commenced to contain scientific articles apart from reports of transactions of various sections of the society.

[4] Under the society's auspices national conferences were held at Turin (1898), Leghorn (1901), Naples (1903), Florence (1908), Padua (1909), Genoa (1912), Trieste (1919) and Naples (1921).

analysis situs, mathematical logic, research regarding axioms). Most of the articles are in French, and the use of languages other than English, French, Italian and German is disallowed.

The first volume opens with a portrait frontispiece, and a brief sketch, of ZYGMUNT JANISZEWSKI, the founder of the journal, and a member of the editorial board. He was born at Warsaw July 12, 1888, became doctor de l'Université de Paris in 1911, maître de conférence in mathematics at the University of Leopold in 1913, and professor at the University of Warsaw in 1919. He died January 3, 1920. A list of his publications is given. The volume contains also 24 articles or notes by 8 authors, and 10 proposed problems.

In the second volume there are 30 articles or notes by 13 authors, and 6 more problems. The last article, 256–285, "Sur les correspondances entre les points de deux espaces," is by H. Lebesgue.

The chief editors of the volumes are STEFAN MAZURKIEWICZ and WACLAW SIERPINSKI, professors of mathematics at the University of Warsaw. The periodical is excellently printed on good paper, royal octavo size. Subscriptions to the volumes (15 French francs) may be sent to *Fundamenta Mathematicae*, Mathematical Seminary, University of Warsaw, Warsaw, Poland.

In 1912 it was arranged that the collected papers of Sophus Lie should be assembled under the general direction of Friedrich Engel, and published by Teubner in about seven large volumes. On account of the great increase in cost of publication, the undertaking would have fallen through had not the Norwegian mathematical union (Norsk Matematisk Forening) laid the matter before the committee on the research fund of three million crowns voted by the Norwegian Storting in 1919. As Engel writes, in the last number of the *Jahresbericht der deutschen Mathematiker Vereinigung*, Bäcklund of Lund, Bianchi of Pisa, Hjelmslev of Copenhagen, Klein of Göttingen, Study of Bonn, Veblen of Princeton, and Vessiot of Paris, united in stating that the publication of the collected papers of Sophus Lie was not only highly desirable but also necessary. As a result, the committee of the Storting fund voted 5000 crowns a year for four years, beginning with 1921, to assist in carrying the undertaking to a successful conclusion, which is now assured. Professor Poul Heegaard, of the University of Christiania, is to be one of the editors. It is expected that the first published volume, the third of the set, will be issued during 1921. The tentative title page of the edition is: SOPHUS LIE, *Gesammelte Abhandlungen, im Auftrage des Norwegischen Mathematischen Vereins und mit Unterstützung der Akademien zu Kristiania und Leipzig herausgegeben von Friedrich Engel und Poul Heegaard.*

ARTICLES IN CURRENT PERIODICALS.

BULLETIN OF THE AMERICAN MATHEMATICAL SOCIETY, volume 27, no. 5, February, 1921: "The thirteenth regular meeting of the southwestern section" by L. Ingold, 197–200; "A remark on skew parabolas" by G. Loria, 201; "The pseudo-derivative of a summable function" by W. L. Hart, 202–211; "Note on minimal varieties in hyperspace" by C. L. E. Moore, 211–216; "Notes on electrical theory" by H. Bateman, 217–225; Review by D. N. Lehmer of E.

Lebon's *Table de Charactéristiques de Base 30030, donnant en un seul coup d'œil les facteurs premiers des nombres premiers avec 30030 et inférieurs à 901,800,900.* Tome I, Premier Fascicule (Paris, 1920), 225–227; Reviews by D. E. Smith, of G. Loria's *Newton* (Rome, 1920), 228–230, and of T. L. Heath's *Archimedes* (London, 1920), 232–234; Reviews by R. D. Carmichael of W. Ahrens's *Mathematiker Anekdoten* (2d edition, Leipzig and Berlin, 1920), 230–231, and of G. Giraud's *Leçons sur les Fonctions automorphes* (Paris, 1920), 231–232; Review by V. Snyder of J. L. S. Hatton's *The Theory of the Imaginary in Geometry together with the Trigonometry of the Imaginary* (Cambridge, 1920), 234–236; "Comment on a previous review" [of Eddington's *Space, Time, and Gravitation*, by E. B. Wilson] by C. N. Moore, 236; Notes, 237–241; New Publications, 242–244—No. 6, March: "The twenty-seventh annual meeting of the American Mathematical Society" by R. G. D. Richardson, 245–265; "Reciprocal subgroups of an abelian group" by G. A. Miller, 266–272; "Proof of an arithmetic theorem due to Liouville" by E. T. Bell, 273–275; "A sequence of polynomials connected with the nth roots of unity" by T. H. Gronwall, 275–279; "The minimum area between a curve and its caustic" by P. R. Rider, 279–284; Review by L. W. Dowling of R. Marcolongo's *Il Problema dei Tre Corpi da Newton ai Nostri Giorni* (Milan, 1919), 284–285; Review by V. Snyder of T. Schmid's *Darstellende Geometrie*, volume 1, second edition (Berlin and Leipzig, 1919), 285; Notes, 286–289; New Publications, 289–292—No. 7, April: "The Chicago meeting of the American Mathematical Society" by A. Dresden, 293–308; "Pleasant questions and wonderful effects" [Presidential address delivered before the American Mathematical Society, December 28, 1920] by F. Morley, 309–312; "Fallacies and misconceptions in Diophantine analysis" by L. E. Dickson, 312–319; "On the Fourier coefficients of a continuous function" by T. H. Gronwall, 320–321; "Extension of an existence theorem for a non-self-adjoint linear system" by H. J. Ettlinger, 322–325; "On the Cauchy-Goursat theorem" by R. L. Borger, 325–329; "On a general arithmetic formula of Liouville" by E. T. Bell, 330–332; Review by L. W. Dowling of O. S. Adams's *General Theory of Polyconic Projections* (Washington, United States Coast and Geodetic Survey, 1919, special publication no. 57), 332–333; Review by G. A. Pfeiffer of A. Fraenkel's *Einleitung in die Mengenlehre* (Berlin, 1919), 333–334; Review by E. B. Cowley of E. Kenison and H. C. Bradley's *Descriptive Geometry* (New York, 1917), 334–335; Notes, 336–337, New Publications, 337–340.

L'Education Mathématique, volume 18, January, 1921: "Sur la relation d'Euler" by J. Coissard, 57–58—February: "Sur les cercles d'Apollonius d'un triangle" by A. Julson, 65–66, 73–74.

Financier, New York, volume 116, March 15, 1921: "Is banking a science?" by C. C. Grove, 10–11, 71–73.

Isis, volume 3, no. 1, January, 1920: "The purpose of Zeno's arguments on motion" by F. Cajori, 7–20—No. 2, September: "Did Fermat have a solution of the so-called Pellian equation?" by J. M. Child, 255–262 [Last sentence: "Hence I conclude that Fermat did have a general proof, and that it was substantially what I have given."].

Journal of the Indian Mathematical Society, volume 12, October, 1920: "An algebra of arithmetical functions" (continued) by F. Hallberg, 161–169; "Double points and lines" (concluded) by B. Rao, 170–174; "Invariants of a conic" by G. A. Srinivasan, 175–177; Problems and Solutions, 178–200—December: "An algebra of arithmetical functions" (continued) by F. Hallberg, 201–216; "Singularities of pedal curves" by A. Narasinga Rao, 217–222; Problems and Solutions, 223–240.

Nature, volume 106, February 3, 1921: "Mathematical text-books," 722–723 [review of H. T. H. Piaggio's *An Elementary Treatise on Differential Equations and their Applications* (London, 1920), of C. V. Durell and G. W. Palmer's *Elementary Algebra, Part i* (London, 1920), of R. E. Moritz's *A Short Course in College Mathematics* (New York and London, 1919) and of F. W. Dobbs and H. K. Marsden's *Arithmetic, Part ii* (London, 1920)]—Volume 107, March 3: "Mathematical papers of Huygens" by J. L. E. D[reyer], 4–5 [review of *Œuvres Complètes de Christian Huygens*, vol. 14]—March 10: "Relativity and the velocity of light" by C. O. Bartrum and J. H. Jeans, 42–43; "Relativity and the deviation of spectral lines" by H. J. Priestley, 43—March 24: Review of A. L. Bowley's *Elements of Statistics* (4th edition, London, 1920), 102–103.

Philosophical Magazine, sixth series, volume 41, February, 1921: "Notes on times of descent under gravity, suggested by a proposition of Galileo's" by W. B. Morton and T. C. Tobin, 225–239 [Proposition 36 of the *Dialogues:* A particle which slides to the lowest point O of a vertical circle starting from rest at any point B of the circumference below the level of the center, will make the journey in a shorter time if it moves along two successive chords BA, AO of the circle than if it goes directly along the one chord BO].

PRINTING ART, volume 37, no. 2, April, 1921: "Golden proportions in design" by F. T Singleton, 133–144.

REVUE DE L'ENSEIGNEMENT DES SCIENCES, volume 14, November–December, 1920: "Formules relatives à l'ellipse et à l'hyperbole" by G. Fontené, 193–196; "Sur la composition des vitesses et la composition des accélérations" by P. Montel, 196–199; "Sur deux théorèmes relatifs aux courbes planes algébriques et aux surfaces algébriques" by R. Malloizel, 199–203; "Sur le calcul de π au moyen d'expressions analogues à $4 \operatorname{arctg} \frac{1}{5} - \operatorname{arctg} \frac{1}{239}$" by A. Lévy, 203–210; "Sur les sommes des puissance semblables des n premiers nombres entiers" by H. Girard, 210–212; "Intersection d'une droite et d'une hyperbole" by J. Angelloz-Pessey, 213–214; "Sur un procédé de Fermat" by J. Angelloz-Pessey, 214–215; "Polaire d'un point par rapport à un cercle" by J. Lemaire, 215; "Note de géometrie descriptive" by J. Juhel-Renoy, 216; "Sur le trapèze harmonique" by C. Michel, 217–218; "Sur l'équation différentielle linéaire du second ordre à coefficients constants" by C. Michel, 218–221; "Examens et concours de 1920," 221–233; Index, 235–236.

REVUE DE MATHÉMATIQUES SPÉCIALES, volume 13, April, 1921: "Sur les hélices cylindriques" by M. Anzemberger, 441–444.

REVUE GÉNÉRALE DES SCIENCES, volume 32, February 28, 1921: "Georges Humbert" by R. d'Adhémar, 97–98 [First paragraph: "Georges Humbert vient de mourir et tous ceux qui l'ont connu regrettent, en lui, le gallant homme, amiable, serviable, au regard vif et bienveillant, et le géomètre éminent, dont l'œuvre est d'une clarté éblouissante et bien française." (See also *1921*, 237)].

REVUE SCIENTIFIQUE, volume 59, February 12, 1921: "Le mathématicien Georges Humbert" by G. Lemoine, 88–89.

SCHOOL SCIENCE AND MATHEMATICS, volume 21, March, 1921: "Mathematical ability as related to general intelligence" by B. R. Buckingham, 205–215; Problems and solutions, 280–284.

SCIENCE, new series, volume 53, April 1, 1921: "National temperament in scientific investigations" by R. D. Carmichael, 298–301 [Fourth paragraph from the end: "Of the three countries which have led in scientific development it seems to be the impartial verdict of history that we owe to France the largest number of works perfect in form and substance and classical for all time; that the greatest bulk of scientific work, at least in more recent decades, has been produced in Germany; but that the new ideas which have fructified science, in earlier times and also in the nineteenth century, have arisen more frequently in Great Britain than in any other country."]; "The American Association for the Advancement of Science, Section L—History of Science sessions" by F. E. Brasch, 315–318—April 22: "Sherburne Wesley Burnham, 1838–1921" by E. B. Frost, 373–377 [see this MONTHLY, *1921*, 236–237].

SCIENTIA, volume 29, March, 1921: "Les contributions des différents peuples au développement des mathématiques. Ière partie: Evènements mémorables et hommes représentatifs dans l'histoire des mathématiques" by G. Loria, 169–184; Review by G. Scorza of L. D. Weld's *Theory of Errors* (New York, 1916) and J. Hadamard's *Four Lectures on Mathematics delivered at Columbia University* (New York, 1915), 220–221.

SCIENTIFIC AMERICAN, volume 124, February 19, 1921: "The new concepts of time and space. A brief account of what Einstein has done to these fundamental notions" by M. Francis, 146–147, 155, 157—March 12, 1921: "The Einstein prize winner," 207 [Lyndon Bolton, autobiographical sketch and portrait; "I was born in Dublin in 1860, but I have lived in England since 1869. My family belonged to the landed gentry class, but I owe nothing to wealth or position. . . . After taking my degree (at Cambridge) in 1883 as a wrangler, I taught science and mathematics at Wellington College but I was attracted by what I had heard of the Patent Office and I entered it through open competition in 1885" (compare *1921*, 191)].

SCIENTIFIC AMERICAN MONTHLY, volume 3, March, 1921: "Leonardo da Vinci as an inventor. Remarkable achievements in sciences and invention of the great Italian artist" by A. A. Hopkins, 263–265.

SCIENTIFIC MONTHLY, volume 12, no. 4, April, 1921: "The history of physics" by H. A. Bumstead, 289–309 [First paragraph: "The beginnings of anything like a connected history of the science which is now called physics may be placed with considerable definiteness about the beginning of the 17th century and associated with the great name of Galileo. It is of course

true that innumerable isolated facts had been known for many centuries which are now included among the data of this science; and many tools and simple machines which are now regarded as applications of physical principles had been devised and used. Even prehistoric man knew some of these—to his very great advantage. But, with one important exception which will be mentioned later, there was, in the ancient world, no connected body of knowledge in this field which can properly be called scientific. In this respect physics differs radically from mathematics, or astronomy, natural history, or medicine, each of which began its modern career with a store of scientific knowledge that had been obtained and put in order before the Renaissance."]; "Perfect and amicable numbers" by L. E. Dickson, 349–354 [First paragraph: "The two types of numbers in our title have had a continuous history extending from the early Greeks to date, and may be justly called the most human of all numbers. To them were early attributed certain social qualities, and later also ethical import, while mystics of the middle ages believed that they possessed special powers as talismans. Continuously for twenty centuries a wide-spread interest has been taken in the purely numerical questions and puzzle problems which arose in the study of these remarkable numbers. We shall present here the more essential facts and fancies in the quaint history of these most human of all numbers."].

SPHINX-ŒDIPE, volume 15, October, 1920: "Sur le folium double" by G. Loria, 1–4—November: "Sur les nombres qui sont, de n façons, la somme de trois bicarrés" by R. Goormaghtigh, 161–162; "Une illusion de Fermat" by L. Aubry, 162–163—December: "Résumé de la conférence générale de M. L. E. Dickson," 177–178; "Errata dans les tables de facteurs linéaires des formes $x^2 \pm Dy^2$" by P. Poulet, 180–181—Volume 16, February, 1921: "Sur les carrés d'Euler" by A. Margossian, 17–21; "Deux nombres curieux" by V. Thebault, 21 [$567^2 = 321489$ and $854^2 = 729316$]; "Au sujet d'une description du folium double due à J. B. Suardi et retrouvée par M. Gino Loria" by R. Goormaghtigh, 22–23—March: "Notice sur Charles Ange Laisant" (à suivre), 33–36; Review by H. Brocard of R. C. Archibald's "Notes on the logarithmic spiral, golden sections and the Fibonacci series" (New Haven, Conn., 1920).

THE TIMES LITERARY SUPPLEMENT, London, volume 19, December 30, 1920: "Correspondence—First use of the decimal point" by J. D. White, 891 [The letter: "Sir,—With reference to my letter on 'The First Use of the Decimal Point,' in *The Times Literary Supplement* of September 9, [cf. this MONTHLY, *1921*, 83], Professor Florian Cajori, of Colorado College, has kindly written to me, quoting from the Second Edition of his *History of Mathematics* (1919) this passage:

"'Historians of Mathematics do not yet agree to whom the first introduction of the decimla point or comma should be ascribed. Among the candidates for the honour are Pellos (1492), Bürgi (1592), Pitiscus (1608, 1612), Kepler (1616), Napier (1616, 1617). This divergence of opinion is due mainly to different standards of judgment. If the requirement made of candidates is not only that the decimal point or comma was actually used by them, but that they must give evidence that the numbers were actually decimal fractions, that the point or comma was with them not merely a general symbol to indicate a separation, that they must actually use the decimal point in operations including multiplication or division of decimal fractions, then it would seem that the honour falls to John Napier, who exhibits such use in his 'Rabdologiæ,' 1617. Perhaps Napier received the suggestion for this notation from Pitiscus, who, according to Eneström, uses the point in his 'Trigonometria' of 1608 and in 1612, not as a regular decimal point, but as a more general sign of separation.'

"Francis Pellos, the first of the writers thus mentioned, was the author of a work in the dialect of Nice, entitled 'Sen Segue de la Art de Arithmeticha,' which was published at Turin in 1492—the year that Columbus discovered America, and within fifty years of the first printing from movable types. An examination of the copy of this rare work in the British Museum Library shows that on pp. 10–12 Pellos gives the following, among other examples, of the method of division 'per una figura' and 'per desenals he centenals.' In the first case the divisor, dividend, and quotient are arranged as here shown, and I have set out the others in the same form, the figures, of course, being as in the original work:

$5)5789657$	$30)583604.3$	$400)78965.73$
$1157931\frac{2}{5}$	$19453\frac{2}{3}\frac{3}{0}$	$197\frac{1}{4}\frac{7}{0}\frac{3}{0}$

"It will be seen that Pellos—as he explains in the text—uses the point for marking off from the dividend as many figures as there are 0's at the end of the divisor, so as to assimilate, for instance, division by 400 to division by 4; but he can hardly be said to use it as a true decimal point for separating the integers from the decimals in the expression of a quantity, and he states the remainder as a fraction of the original divisor.

"Stevinus introduced the operative use of decimals in 1585, and they were also used operatively by Bürgi in 1592, and Beyer in 1603. Both Stevinus and Beyer indicated the last integer and the first, second, &c., decimal figures by placing after or above them as exponents the numbers 0, 1, 2, &c. (a notation probably suggested by that which had long been used for sexagesimal fractions and survives in the marks for minutes and seconds), and even Bürgi, who thought it sufficient to place an 0 underneath the last integer, failed to attain 'the elegant simplicity' of the decimal point.

"Pitiscus, as shown in my previous letter, appears to have been the first to use the decimal point for separating the integers from the decimals in expressions of quantity. But he does not appear to have used it operatively; the evidence indeed is rather the other way. In the tables to both the second and the third editions of his 'Trigonometria' (1608, 1612) he gives, for instance, the sine of 30° 20′ to radius 100,000 as 50502.98; but in the example (at the beginning of book 3 in both these editions) of ascertaining the length of the sine of this angle for a radius of 24 ft., he takes the sine as 5050298 to radius 10,000,000, and states the result as $12\frac{1207152}{10000000}$.

"The decimal point was not used in the original Latin edition of Napier's 'Descriptio' of 1614, but it was used in the English edition of 1616, as described in my previous letter. In neither edition, however, is there an example of its operative use. But in Napier's 'Rabdologiæ,' Edinburgh, 1617, bk. 1. ch. 4, the description of the use of the numbered rods, commonly called Napier's Bones, in compound division is followed by an 'Admonitio pro Decimali Arithmetica,' in which Napier refers to Stevinus's Decimal Arithmetic, and says—to quote the translation in Mr. Macdonald's book mentioned below—

"'Since there is the same facility in working with these fractions as with the whole numbers, you will be able, after completing the ordinary division, and adding a period or comma, as in the margin, to add to the dividend or to the remainder one cypher to obtain tenths, two for hundredths, three for thousandths, or more afterwards as required; and with these you will be able to proceed with the working as above' . . .

"In the margin is the example of dividing 1180 by 432 (sic), with the result stated as 1993,273 which, as he explains, is equivalent to $1993\frac{273}{1000}$ or 1993, 2′ 7″ 3‴. This is probably the earliest printed instance of the operative use of decimals with the decimal point, or its equivalent, the decimal comma.

"In Napier's 'Constructio,' with Notes by Briggs, which was printed in 1619 after Napier's death, decimals are both described and used operatively with the decimal point, strictly so called. Mr. W. R. Macdonald, in his translation with notes (1889) of the 'Constructio,' makes special reference to this feature on pp. 88, 89, setting out the above quoted passage from 'Rabdologiæ' (1617), and observing that it is interesting as Napier's first published reference to decimal arithmetic, though sections 4, 5, and 47 of the 'Constructio' which deal with it 'must have been written long before that date.'"]

UNDERGRADUATE MATHEMATICS CLUBS.

All reports of club activities should be sent to E. L. Dodd, 3012 West Ave., Austin, Texas

CLUB ACTIVITIES.

THE MATHEMATICS CLUB OF ALBION COLLEGE, Albion, Michigan.
[*1918*, 354; *1919*, 409.]

Officers, elected each semester in 1919–20 and 1920–21, were as follows: Presidents, Esther Pearl '20, Joyce Hadaway '20, Elizabeth Gordon '21, Gertrude Pratt '21; vice-presidents, Joyce Hadaway '20, Harland Hatch '20, Gertrude Pratt '21, Mary Hutchins '21; secretary-treasurers, Almira Priest '20, Almira Priest '20, Mary Hutchins '21, Harold Black '23; third member of program committee, Elizabeth Gordon '21, Clark Dean '21, Christine Niemann '21, Marie de Vries '21.

The programs, which are listed below, usually included a general consideration of some topic at roll call, followed by one or more papers or talks.

October 28, 1919: "Our great inheritance—Mathematics" by Esther Pearl '20.

November 4: "Graphical algebra" by Carl Anderson '21; "Mathematical lists" by Elizabeth Gordon '21.

December 2: The work of great mathematicians; "The Flatlanders" by Gertrude Pratt '21.

December 9: Myths of constellations; talk by Mary Hutchins '21.

December 16: Applications of mathematics; "Geometrical representations of intermediate forms" by Donald Herrick '21.

January 14, 1920: Women in mathematics; "Hyperbolic functions" by Esther Pearl '20.

January 20: Letters from alumnæ read by Almira Priest '20.

January 27: Mathematical publications; "Mathematical fallacies" by Carl Anderson '21; "History of mathematics in the United States during the nineteenth century" by Christine Niemann '21.

March 2: Historical theorems; "The Leibnitz-Newton controversy" by Charlotte LaMastus '21 and Gertrude Pratt '21.

March 16: Present day mathematicians; "Graphs in ninth-grade algebra" by Elizabeth Gordon '21; "Eulerian integrals and Gamma functions" by Clark Dean '21.

April 13: Difficult solutions in algebra; "The junior high school" by Joyce Hadaway '20; "Historical development of the number system" by Clark Dean '21.

April 27: Trigonometric functions and geometrical constructions; "Elementary algebra and geometry" by Professor Head.

May 11: "Harmonics" by Joyce Hadaway '20; "Anharmonics" by Miss Hubert.

November 2: Facts about mathematicians; "A 'flu' dream in mathematics" by Gertrude Pratt '21; "The project method in teaching" by Mary Hutchins '21.

November 16: Mathematical jokes; "Applications of the forms of zero and unity" by Marie Plumb '22; "The circles of Appollonius" by Christine Niemann '21.

November 30: Constellations and myths about them; "Calculation by geometry of astronomical distances" by Elizabeth Gorden '21; "Mathematics, the farmer, and the weather" by Charlotte LaMastus '21.

December 21: Trigonometric functions and their development; "The math quest" by Harold Black '23; "Hyperbolic functions" by Gertrude Pratt '21.

January 18, 1921: Mathematical current events; "The Courtis intelligence test" by Rex Miller '21; "Problems in junior high school mathematics" by Lona Stockmeyer '21.

January 25: "Minimum requirements for high school mathematics" by Marie deVries '21; "The monkey and cocoanut problem" by Professor E. R. Sleight.

(Report by Mr. Black.)

MU THETA EPSILON, ALPHA CHAPTER, University of California, Berkeley, Cal.

A mathematics club was organized at the University of California under the name of Mu Theta Epsilon in April, 1920, with three aims: (*a*) The stimulation of interest in mathematics and the inducement of better work therein; (*b*) The discussion of mathematical topics; (*c*) The promotion of congeniality among students and closer coöperation between students and professors.

Membership is limited to women students, and includes juniors, seniors, and graduates. Twenty-two active members have participated during 1920–21, and regular monthly meetings have been held.

The officers for the current year are: President, Mamie Cohen '20; vice-president, Evelyn Aylesworth '20; secretary, Helene Clark, '21; treasurer, Nellie Bartlett '20; active faculty member, Dr. Pauline Sperry.

The following programs have been given:

September 1, 1920: "A special quartic curve" by Elsie McFarland '17; "Devices in mathematics used by peasants of Russia" by Professor B. A. Bernstein.

October 6: "Mathematics clubs in high schools" by Constance Kendall '20; "Geometry of the circle or compass" by Gladys-Mary Campbell '18.

November 3: "Mathematical theory of the satine arrangement" by Mildred Hurd '21; "The fourth dimension" by Ruth Brant '21.

December 1: "A discussion of hyperspace" by Nina Alderton '14; "Mathematical fallacies" by Nellie Bartlett '20.

February 1, 1921: "Concrete multipliers" by Lora Lind '21.

March 2: "Lambert's solution of the equation by infinite series" by Mamie Cohen '20; "D. M. D. Method of computing areas" by Thelma Hansen '21.

The Beta Chapter of Mu Theta Epsilon was established at the University of Southern California in Los Angeles in December, 1920. Plans for the organization of chapters at Leland Stanford University at Palo Alto, and Pomona College at Claremont, are under way. It is hoped to make Mu Theta Epsilon a national mathematics honor society in the near future.

(Report by Miss Clarke.)

THE JUNIOR MATHEMATICAL CLUB OF THE UNIVERSITY OF CHICAGO, Chicago, Ill.

[*1918*, 34, 448.]

In the autumn of 1918, owing to the great confusion in connection with S. A. T. C., the club was not organized, but in the autumn of 1919 its work was again resumed.

The officers for 1919–20 were as follows: President, Ernest Zeisler Gr.; secretary and treasurer, Lila Nelson Gr.; program committee, Mrs. Mayme I. Logsdon Gr., Gladys Freeman Gr., and J. W. Lasley Gr. The number of members for the year was forty-two, and the average attendance, thirty-one.

The programs for 1919–20 were as follows:

October 22, 1919: "History and purposes of the Club" by Professor H. E. Slaught.

November 5: "Methods of counting" by Mrs. Mayme I. Logsdon Gr.

November 26: "Isosceles triangles" by Isaac Schour '21; "Magic squares" by Leah Libman '21; "Game of kim" by Lila Nelson Gr.

December 3: "The isosceles triangle" by C. C. MacDuffee Gr.; "Zeno's paradoxes of motion" by V. D. Gokhale Gr.

December 17: "Non-euclidean geometry" by Roy Haskell Gr.

January 14, 1920: "Arithmetical properties of numbers" by Gladys Freeman Gr.

February 11: "Some simple examples of covariants and invariants" by J. W. Lasley Gr.

February 25: "Concerning two particular matrices" by Professor E. H. Moore.

March 10: "Hyperbolic functions" by E. T. Browne Gr.

April 7: "Solution of quadratics" by J. C. Kamplain Gr.; "Interpolations" by S. J. Jacobson Gr.

April 21: "The theory of the complex number system" by C. C. MacDuffee Gr.

May 5: "The fundamental theorem of algebra" by Gladys Freeman Gr.

May 19: "The philosophical conception of the universe" by Dr. Edward Lasker, efficiency engineer, Chicago, Ill.

June 9: A beach party was given for the Graduate Mathematical Club and the Faculty of the department.

The officers for 1920–21 were as follows: President, E. T. Browne Gr.; vice-president, E. B. Miller Gr.; secretary and treasurer, Agnes Jones Gr.; program committee, Claribel Kendall Gr., E. T. Browne Gr., Myrtle Collier Gr. The number of members for the year was twenty-five, and the average attendance, twenty-one.

The programs for 1920–21 were as follows:

November 10, 1920: "History, purposes and aim of the Club" by Professor Slaught.

November 24: "Hyperbolic functions" by E. B. Miller Gr.

December 8: "The history of the parallel postulate" by Claribel Kendall Gr.

January 5, 1921: "Exterior ballistics" by Major H. E. Miner, U. S. Army.

January 19: "Linear algebras" by C. C. MacDuffee Gr.

February 2: "Zermelo's theorem: 'Every class can be well ordered'" by V. D. Gokhale Gr.

February 16: "Two famous geometry problems of antiquity" by Roy Haskell '21.

(Report by Miss Jones.)

PROBLEMS AND SOLUTIONS.

EDITED BY B. F. FINKEL, OTTO DUNKEL, AND H. P. MANNING.

Send all communications about Problems and Solutions to B. F. FINKEL, Springfield, Mo.

PROBLEMS FOR SOLUTION.

[N.B. Problems containing results believed to be new, or extensions of old results are especially sought. The editorial work would be greatly facilitated if, or sending in problems, proposers would also enclose any solutions or information that will assist the editors in checking the statements. In general, problems in well-known text-books, or results found in readily accessible

sources, will not be proposed as problems for solution in the MONTHLY. In so far as possible, however, the editors will be glad to assist members of the Association with their difficulties in the solution of such problems.]

2908. Proposed by L. E. DICKSON, University of Chicago.

If f is a homogeneous polynomial in n variables and H is its Hessian determinant, prove that the Hessian of f^2 is cHf^n, where c is a constant.

2909. Proposed by J. S. STROMS, University of Maine.

A pinochle pack contains 48 cards, eight each of aces, kings, queens, jacks, tens, and nines. When three play, they are distributed by giving fifteen to each player and leaving three in the "kitty." When one holds two jacks of diamonds and two queens of spades in the same hand he has what is called double pinochle. When he holds eight aces he wins the game with 1000 points. What are the chances (*a*) of getting double pinochle; (*b*) of getting eight aces?

2910. Proposed by DANIEL KRETH, Wellman, Iowa.

The segments formed on the base of a triangle by the perpendicular from the opposite vertex are m and n. The product of the other two sides is p. Compute the two unknown sides and give a simple construction for the triangle.

2911. Proposed by D. A. ABRAMS, Chicago, Ill..

In the analysis of stresses in a flat slab of reinforced concrete, the following integral arises: $\int_0^x \int_0^y \int_0^z dxdydz$, where the upper limits for x, y, z are b, a, and $wb^3y^3(a-y)^3/[a^3x^3(b-x)^3 + b^3y^3(a-y)^3]$. Evaluate this integral.

2912. Proposed by T. W. JACKSON, Jamestown College, N. D.

Given c, the chord of a circle, determine r, the radius, so that $3c$ is equal to the major arc of the circle.

2913. Proposed by PAUL CAPRON, U. S. Naval Academy.

Given

$$S(z) \equiv nz\left[1 + \sum_1^\infty (-1)^p \frac{(n^2-1)(n^2-3^2)\cdots\{n^2-(2p-1)^2\}}{(2p+1)!} z^{2p}\right]$$

$$C(z) \equiv 1 + \sum_1^\infty (-1)^p \frac{n^2(n^2-2^2)(n^2-4^2)\cdots\{n^2-(2p-2)^2\}}{(2p)!} z^{2p}$$

show that, for any value of n, $|\cos x| < 1$, $|\sin x| < 1$:

$$\sin(nx) \equiv S(\sin x) \equiv \sin\left(\frac{n\pi}{2}\right) C(\cos x) - \cos\left(\frac{n\pi}{2}\right) S(\cos x);$$

$$\cos(nx) = C(\sin x) \equiv \cos\left(\frac{n\pi}{2}\right) C(\cos x) + \sin\left(\frac{n\pi}{2}\right) S(\cos x).$$

2914. Proposed by HARRIS HANCOCK, University of Cincinnati.

If $a_1, a_2, \cdots, a_n$ are n positive integers, a_{ij} the greatest common divisor of a_i and a_j, d_m the greatest common divisor of all products of every m of these numbers ($m = 1, 2, \cdots, n-1$), then is

$$\prod_{ij} a_{ij} = d_1 d_2 \cdots d_{n-1}; \quad (j > i;\ i = 1, 2, \cdots, n;\ j = 2, 3, \cdots, n).$$

In general show that this theorem is true if $A_1, A_2, \cdots, A_n$ are any functions integral in any number of variables, with rational integral coefficients, or with algebraic integral coefficients. [Remark: This is a generalized statement in positive rational integers of the following theorem of much importance in the theory of algebraic numbers and due to Dedekind (Dirichlet, *Zahlentheorie*, supplement XI): Let **A**, **B**, **C** be three moduls (Dedekind). Denote the greatest common divisor of **A** and **B** by **A** + **B** and their product by **A**·**B**. Dedekind proves that

$$(\mathbf{A}+\mathbf{B})(\mathbf{B}+\mathbf{C})(\mathbf{C}+\mathbf{A}) = (\mathbf{A}+\mathbf{B}+\mathbf{C})(\mathbf{AB}+\mathbf{BC}+\mathbf{CA}).$$

Denote the greatest common divisor of two moduls $\mathfrak{A}_i$ and $\mathfrak{A}_j$, that is, $\mathfrak{A}_i + \mathfrak{A}_j$ by $\mathfrak{A}_{ij}$; and write

$$\mathfrak{D}_1 = \mathfrak{A}_1 + \mathfrak{A}_2 + \cdots + \mathfrak{A}_n, \qquad \mathfrak{D}_2 = \mathfrak{A}_1\mathfrak{A}_2 + \mathfrak{A}_1\mathfrak{A}_3 + \cdots + \mathfrak{A}_{n-1}\mathfrak{A}_n,$$
$$\mathfrak{D}_3 = \mathfrak{A}_1\mathfrak{A}_2\mathfrak{A}_3 + \mathfrak{A}_1\mathfrak{A}_2\mathfrak{A}_4 + \cdots + \mathfrak{A}_{n-2}\mathfrak{A}_{n-1}\mathfrak{A}_n, \cdots$$
$$\mathfrak{D}_{n-1} = \mathfrak{A}_2\mathfrak{A}_3 \cdots \mathfrak{A}_n + \mathfrak{A}_1\mathfrak{A}_3 \cdots \mathfrak{A}_n + \cdots + \mathfrak{A}_1\mathfrak{A}_2 \cdots \mathfrak{A}_{n-1}.$$

Show that

$$\mathfrak{A}_{12}\,\mathfrak{A}_{13} \cdots \mathfrak{A}_{n-1,\,n} = \mathfrak{D}_1\mathfrak{D}_2 \cdots \mathfrak{D}_{n-1}.]$$

2915. Proposed by HARRIS HANCOCK, University of Cincinnati.

Determine x so as to satisfy the two congruences $3x^2 \equiv 0\ (3N)$, $x^3 + a \equiv 0\ (9N^2)$, where $a = N^2 \cdot n$, and the two integers N, n have no common factor, and neither contains a squared factor.

2916. Proposed by HARRIS HANCOCK, University of Cincinnati.

If p is any rational prime integer, and if $\alpha(\neq 1)$ is any root of $x^p - 1 = 0$, show that $p = \mathfrak{P}_1 \cdot \mathfrak{P}_2 \cdots \mathfrak{P}_{p-1}$, where $\mathfrak{P}_i$ $(i = 1, 2, \cdots, p-1)$ are the ideals $(p, 1 - \alpha^i)$, which in turn may be reduced to principal ideals. [Remark: This is rather a good elementary example to show that an integer prime in one realm is factorable in a more extended realm.]

2917.

A parabola is rolled upon a fixed right line. Find the locus of (a) its vertex; and (b) its focus.

2918. Proposed by NATHAN ALTSHILLER-COURT, University of Okla.

Find planes which cut four given lines in four concyclic points.

2919. Proposed by V. M. SPUNAR, Chicago, Ill.

An equilateral hyperbola which touches a conic and is concentric with it is called a hyperbolic tangent to the conic. Being given two hyperbolic tangents to a conic, the arc of any third hyperbolic tangent which is intercepted by the first two subtends a constant angle at either focus of the given conic.

NOTES.

19. A Curve of Pursuit. Apropos of Note **10** (*1921*, 184; cf. *1921*, 278, 281), referring to a problem proposed by Lucas in 1877, a very similar problem is given as a worked out exercise in Bateman, *Differential Equations*, 1918, pp. 8–10. It is as follows: "Three boys running at the same speed u chase one another. A pursues B, B pursues C and C pursues A. Find a differential equation which will indicate the way in which the ratios of the sides of the triangle ABC vary." A footnote states that this problem is due to Professor Frank Morley. His differential equation is discussed in a paper by F. E. Hackett, "A numerical solution of the triangular problem of pursuit," in *The Johns Hopkins University Circular*, July, 1908, pp. 135–137.

A. H. WILSON.

20. A Problem in Investment. There has been inquiry concerning the following problem given in a less general form, with a reference to *Engineering News*, volume 48, 1902, pp. 362–363, in E. B. Skinner, *The Mathematical Theory of Investment*, Boston, 1913, p. 140.

Bonds are issued, N in number, of a face value of A each, bearing interest at rate r. At the end of a years and at the end of each year thereafter a certain number of these bonds is to be redeemed at a price which bears the ratio R to the face value A. How many bonds must be redeemed each year in order that the whole issue shall be paid for at the end of n years, and that the sum of the interest

on the unpaid bonds and the amount paid to redeem the bonds shall be the same for each of the $n - a + 1$ years?

This problem is completely solved in *Engineering News* (l.c.).

21. Involutes of a circle and a pasturage problem. On page 128 of *Problems and Solutions. Associateship Examinations, Parts I and II, 1915–1919* (New York, Actuarial Society of America, 1921) is a brief solution of the following problem in the examinations set for 1918: "A circular wall of radius a stands in the middle of a large field. A horse is tethered to the outside of this wall by a rope the length of which is equal to half the circumference of the wall. Show that the area over which the horse can graze is $(5/6)\pi^3 a^2$." The area is evidently composed of a semicircle of radius πa and two areas with arcs of involutes of the given circle as outer boundaries.

A somewhat more complicated problem was proposed over one hundred and seventy years ago, in *The Ladies Diary or the Woman's Almanack*, 1748. The problem was as follows (page 41): "Observing a *Horse* tied to feed in a Gentleman's *Park*, with one End of a Rope to his *Fore-foot*, and the other end to one of the Circular *Iron-Rails*, inclosing a Pond, the Circumference of which Rails being 160 yards, equal to the Length of the Rope, what Quantity of Ground, at most could the Horse feed?" A solution was given on pages 25–26 of the *Diary* for 1749, and the answer found was[1] "76257.86 sq. yards = 15*A*.2*R*.12*P*."

Involutes of a circle may be connected with many other curves. For example: If an involute of a circle, of radius a, rolls on a straight line, the locus of the center of the circle is a parabola whose parameter is $2a$ (J. Clerk Maxwell,[2] 1849)—The locus of the center of the circle (radius a) of an involute, rolling on an orthogonal tractory of the catenary, whose equation is

$$y = \frac{x}{2a}\sqrt{x^2 - a^2} + \frac{a}{2}\log\left(\sqrt{\frac{x^2}{a^2} - 1} + \frac{x}{a}\right),$$

is the axis of y (Maxwell, 1849)—If an involute of a circle rolls on an equal involute with corresponding points in contact, the center of the circle traces a spiral of Archimedes (Maxwell, 1849)—The involute of a circle is the locus of the pole of a logarithmic spiral rolling on a concentric circle (Maxwell, 1849; often attributed to Haton de la Goupillière)—The pedal of an involute with respect to the center of its circle is a spiral of Archimedes (Practically the same as the third of the results by Maxwell; see also Mannheim,[3] 1858)—The caustic by reflection of an involute for rays emanating from the center of its circle is an evolute of a spiral of Archimedes (Haton de la Goupillière,[4] 1863)—The inverse of

[1] See also *The Mathematical Questions proposed in the Ladies' Diary*, edited by T. Leybourn, volume 2, London, 1817, pp. 6–7; *The Diarian Miscellany* by C. Hutton, volume 2, London, 1775, p. 269; *The Diarian Repository*, London, 1774, pp. 507–508.

[2] This and the next three results are taken from the remarkable memoir on "The theory of rolling curves" presented to the Royal Society of Edinburgh when Maxwell was 18 years of age. See the *Scientific Papers of James Clerk Maxwell*, volume 1, 1890, pp. xi, 22, 26, 28.

[3] *Nouvelles Annales de Mathématiques*, 1858, pp. 186–187 and 1860, 186–187.

[4] *Nouvelles Annales de Mathématiques*, 1863, pp. 336, 494–500, 548–550.

an involute with respect to the center of its circle is a spiral tractrix, that is, a curve which, in polar coördinates, has a tangent of constant length (Haton de la Goupillière,[4] 1863)—The centers of curvature for the points of contact of an involute rolling on a straight line is a parabola (Cesàro,[1] 1884)—The points of contact of tangents drawn to an involute from any point of its plane lie on a limaçon of Pascal (Fouret,[2] 1888).

The involute of a circle seems to have been first conceived in 1693 when Huygens was considering clocks without pendulums which might be of service on sea going vessels.[3] In this connection he originated an apparatus in which the involute of a circle plays an essential rôle.

In 1891 it became desirable to install in the Royal Observatory at Greenwich a larger telescope. This necessitated that a larger dome, 36 feet in diameter, be built upon a circular wall 31 feet 4 inches in diameter. The form adopted was that of a "surface generated by the revolution of an involute of a circle, 7 feet in diameter, with its center in the plane of the rail, and 5 feet from the axis, the curve being completed near the apex by an arc of a circle (of 13 feet 3 inches radius) so that it cuts the axis at right angles. The diameter of the dome is 36 feet at a height of 7 feet above the rail."[4] ARC.

SOLUTIONS.

2809 [1920, 80]. Proposed by the late L. G. WELD.

Find the nth term of the series defined by the relation, $u_{i+2} = u_i + u_{i+1}$, in which $u_1 = u_2 = 1$.

SOLUTION BY HENRI SEBBAN, Boufarik, Algeria.

Consider the sequence

$$1, 1, 2, 3, 5, 8, \cdots$$

in which each term beginning with the third is the sum of the two terms immediately preceding. In order to express a term as a function of u_1, u_2 and i, let us set $u_i = Ax_1^i + Bx_2^i$ and endeavor to determine A, B, x_1 and x_2 so as to satisfy the law of the sequence. Then

$$\begin{aligned} Ax_1^i + Bx_2^i &= Ax_1^{i-1} + Bx_2^{i-1} + Ax_1^{i-2} + Bx_2^{i-2} \\ &= Ax_1^{i-2}(x_1 + 1) + Bx_2^{i-2}(x_2 + 1), \end{aligned}$$

and from this it follows that $x_1 + 1 = x_1^2$, $x_2 + 1 = x_2^2$. Hence x_1 and x_2 are the two roots of the equation $x^2 - x - 1 = 0$, for we cannot have $x_1 = x_2$ unless the sequence reduces to a geometric progression, $u_i = Kx^i$ of ratio x which is not the case when $u_1 = u_2 = 1$. It will be seen that in the general case A and B are determined by the initial conditions and are different from zero. Hence we may set

$$x_1 = \frac{1 + \sqrt{5}}{2}, \qquad x_2 = \frac{1 - \sqrt{5}}{2}, \qquad \text{and} \qquad u_i = A\left(\frac{1 + \sqrt{5}}{2}\right)^i + B\left(\frac{1 - \sqrt{5}}{2}\right)^i,$$

where A and B are to be determined from the conditions $Ax_1 + Bx_2 = 1$, $Ax_1^2 + Bx_2^2 = 1$. Hence, we have after certain reductions

$$A = -B = \frac{1}{\sqrt{5}} \qquad \text{and} \qquad u_i = \frac{(1 + \sqrt{5})^i - (1 - \sqrt{5})^i}{2^i\sqrt{5}}.$$

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[1] *Mathesis*, 1884, pp. 233–235.

[2] *Journal de Mathématiques Spéciales*, 1888, p. 261.

[3] *Œuvres Complètes de Christian Huygens*, volume 10, 1905, pp. 514–515.

[4] *Monthly Notices of the Royal Astronomical Society*, volume 51, May 8, 1891, p. 436.

It will be found that for this series the following relations are true:

$$u_{i+2}u_{i-1} - u_iu_{i+1} = \pm 1, \qquad u_i^4 - u_{i-2}u_{i-1}u_{i+1}u_{i+2} = 1, \qquad \text{and} \qquad S_i = u_{i+2} - u_2,$$

where S_i is the sum of the first i terms of the series. This latter series has received the names Fibonacci's and Lamé's, and is the subject of several interesting studies. In the *Journal de Mathématiques Elémentaires*(Vuibert), 44e année, p. 11, it is shown that the sum of the squares of two consecutive terms of Fibonacci's series is a term of the series. In the *Intermédiaire des Mathématiciens*, 1916, pages 64 and 599, it is proved that if p is a prime of the form $10n \pm 3$ it is a divisor of u_{p+1}; if of the form $10n \pm 1$ it is a divisor of u_{p-1}. In *Nouvelles Annales de Mathématiques*, 1884, p. 533, is a proof of the theorem that $(2u + 1)^i - u^{3i} = 1$ provided that the exponents in the expansion are replaced by indices and the resulting terms u_n are terms of Fibonacci's series.[1]

Also solved by NORMAN ANNING, J. P. BALLANTINE, E. B. ESCOTT, H. L. OLSON, E. J. OGLESBY, and LOUIS WEISNER.

2811 [1920, 81]. Proposed by J. L. RILEY, Stephenville, Texas.

Given cube roots of 60, 61, 63 and 64, to find the cube root of 62 by the method of differences.

SOLUTION AND REMARKS BY E. B. ESCOTT, Oak Park, Illinois.

Let u_1, u_2, u_3, u_4, u_5 be a series of functions with equidistant arguments. By the method of differences, the first term of the fourth order of difference of u_1 is

$$u_5 - 4u_4 + 6u_3 - 4u_2 + u_1.$$

Let this equal zero and solve for u_3, whence

$$u_3 = \tfrac{1}{6}[4(u_2 + u_4) - (u_1 + u_5)].$$

If

$$u_1 = \sqrt[3]{60} = 3.91486\ 76412\ -, \qquad u_2 = \sqrt[3]{61} = 3.93649\ 71831, \qquad u_3 = \sqrt[3]{62},$$

$$u_4 = \sqrt[3]{63} = 3.97905\ 72079\ - \qquad \text{and} \qquad u_5 = \sqrt[3]{64} = 4,$$

we have by above formula, $\sqrt[3]{62} = 3.95789\ 16538$. This is true to 7 decimals, the value to 10 decimals being $3.95789\ 16097\ -$.

A more general method is to use Newton's Interpolation Formula, where the intervals are not necessarily equal, *i.e.*,

$$u_x = u_a + (x - a)\cdot\delta'(a, b) + (x - a)(x - b)\cdot\delta''(a, b, c) + \cdots, \tag{1}$$

where

$$\delta'(a, b) = \frac{u_a - u_b}{a - b}, \qquad \delta''(a, b, c) = \frac{\delta'(a, b) - \delta'(b, c)}{a - c}, \qquad \cdots.$$

See Thiele, *Interpolationsrechnung*, page 5, or Boole, *Finite Differences*, Chapter 3, Exercise 19. In this problem, $u_{62} = \sqrt[3]{62}$, $u_a = u_{60} = \sqrt[3]{60}$, $u_b = u_{61} = \sqrt[3]{61}$, $u_c = u_{63} = \sqrt[3]{63}$, $u_d = u_{64} = \sqrt[3]{64}$.

Then

$$\delta'(61, 60) = \frac{\sqrt[3]{61} - \sqrt[3]{60}}{61 - 60} = .02162\ 95419,$$

$$\delta'(63, 61) = \frac{\sqrt[3]{63} - \sqrt[3]{61}}{63 - 61} = .02128\ 00124,$$

$$\delta'(64, 63) = \frac{\sqrt[3]{64} - \sqrt[3]{63}}{64 - 63} = .02094\ 27921,$$

$$\delta''(63, 61, 60) = \frac{\delta'(63, 61) - \delta'(61, 60)}{63 - 60} = -.00011\ 65098,$$

[1] The literature of this series has been discussed already in this MONTHLY, *1918*, 234–238, 462, by R. C. Archibald; reference to a recent extended elaboration of these notes was made *1920*, 314.—EDITORS.

$$\delta''(64, 63, 61) = \frac{\delta'(64, 63) - \delta'(63, 61)}{64 - 61} = -.00011\ 24068,$$

and

$$\delta'''(64, 63, 61, 60) = \frac{\delta''(64, 63, 61) - \delta''(63, 61, 60)}{64 - 60} = .00000\ 10258.$$

Substituting in Newton's Formula, we have

$$\sqrt[3]{62} = \sqrt[3]{60} + 2 \times .02162\ 95419 + 2 \times 1 \times (-.00011\ 65098) + 2 \times 1 \times (-1) \times .00000\ 10258 \quad (2)$$

$= 3.95789\ 16538$ true to 7 decimals, as in the other solution.

In the above formula, the arguments may be any number, and, in particular, we may have any number of them equal. In this case the "divided differences" become

$$\delta'(a, a) = \lim_{b \doteq a} \frac{u_a - u_b}{a - b} = \frac{du_b}{db}, \qquad \delta''(a, a, a) = \lim_{b \doteq a} \frac{\delta''(a, a) - \delta''(b, b)}{a - b} = \frac{1}{2} \frac{d^2u_b}{db^2},$$

$$\delta'''(a, a, a, a) = \frac{1}{2 \cdot 3} \frac{d^3u_b}{db^3} \cdots.$$

Newton's Interpolation formula for the given functions u_a, u_b, u_c, u_d, u_d, u_d, $\cdots$, where the last function may be used an indefinite number of times, becomes

$$u_x = u_a + (x - a)\delta'(a, b) + (x - a)(x - b)\delta''(a, b, c) + (x - a)(x - b)(x - c) \cdot \delta'''(a, b, c, d) + (x - a)(x - b)(x - c)(x - d) \cdot \delta^{iv}(a, b, c, d, d) + \cdots.$$

In this form, it is a combination of Newton's Interpolation Formula and Taylor's Formula.

In the foregoing problem, using $u_{64} = \sqrt[3]{64}$ three times, we have

$$\delta'(64, 64) = \left(\frac{d\sqrt[3]{x}}{dx}\right)_{x=64} \left(\frac{1}{3} x^{-2/3}\right)_{x=64} = .02083\ 33333,$$

$$\delta''(64, 64, 63) = \frac{\delta'(64, 64) - \delta'(64, 63)}{64 - 63} = -.00010\ 94588,$$

$$\delta'''(64, 64, 63, 61) = \frac{\delta''(64, 64, 63) - \delta''(64, 63, 61)}{64 - 61} = .00000\ 09827,$$

$$\delta^{iv}(64, 64, 63, 61, 60) = \frac{\delta'''(64, 64, 63, 61) - \delta'''(64, 63, 61, 60)}{64 - 60} = -.00000\ 00108,$$

and

$$\delta^{v}(64, 64, 64, 63, 61, 60) = .00000\ 00001.$$

This gives us two more terms in the series (2), viz.,

$$+ 2 \times 1 \times (-1)(-2)(-.00000\ 00108) + 2(1)(-1)(-2)(-2)(.00000\ 00001) = -.00000\ 00440.$$

Then $\sqrt[3]{62} = 3.95789\ 16538 - .00000\ 00440$
$= 3.95789\ 16098$, true to 9 decimals.[1]

[1] The same result may be obtained as follows: In the first part of the discussion above,

$$\Delta_4 = u_5 - 4u_4 + 6u_3 - 4u_2 + u_1$$

was assumed zero, but a correction may be obtained by applying to the terms of Δ_4 the development of u_x in powers of $x - 64$, x being 64 for u_5, 63 for u_4, etc. Then

$$\Delta_4 = u_5^{iv} - 2u_5^{v} + \frac{13}{6} u_5^{vi} - \cdots,$$

where

$$u_5^{(n)} = \frac{d^n \sqrt[3]{x}}{dx^n}\Big]_{x=64}.$$

Hence

$$\sqrt[3]{62} = u_3 = \frac{1}{6}[4(u_2 + u_4) - (u_1 + u_5)] + \frac{\Delta_4}{6}.$$

Note. Newton's Interpolation Formula is equivalent to the well-known Euler-Lagrange Interpolation Formula, but is frequently more convenient to use. As shown in the above example, if the interpolation is not sufficiently accurate, we need only add more terms to Newton's Formula, while the Euler-Lagrange Formula will need to be entirely recomputed.

Also solved by NORMAN ANNING, E. J. OGLESBY, ARTHUR PELLETIER, J. B. REYNOLDS, C. C. WYLIE, and the Proposer.

NOTES AND NEWS.

It is to be hoped that readers of the MONTHLY will coöperate in contributing to the general interest of this department by sending items to H. P. MANNING, Brown University, Providence, R. I.

Associate Professor E. J. MOULTON, of Northwestern University, has been promoted to a full professorship of mathematics.

Professor E. G. BILL, of Dartmouth College, has been appointed to the newly created office, dean of freshmen. He will give up all teaching.

At their Alma Mater, Wellesley College, Miss RACHEL BLODGETT, recently of Radcliffe College, and Miss RUBY WILLIS, have been appointed instructors of mathematics.

At the University of Pennsylvania, Assistant Professors G. G. CHAMBERS and H. H. MITCHELL have been promoted to full professorships.

Mrs. THERON CLARK, of Brown University, has been appointed instructor of mathematics at Bucknell University, Lewisburg, Pa.

Dr. F. D. MURNAGHAN, of The Johns Hopkins University, has been promoted to be an associate professor of applied mathematics.

Associate Professor W. A. WILSON, of Yale University, has been promoted to a full professorship.

Instructors C. S. DOAN and F. H. HODGE, of Purdue University, Lafayette, Ind., have been promoted to assistant professorships.

Mr. L. E. WARD, of Harvard University, has been appointed instructor of mathematics at the Rice Institute, Texas.

Miss MAY B. CARTER, of Brown University, has been appointed instructor of mathematics in The Western College for Women, Oxford, O.

Professor E. R. SMITH, of Pennsylvania State College, has been appointed head of the department of mathematics at Iowa State College, Ames, Ia.

Dr. J. R. RITT, of Columbia University, has been promoted to an assistant professorship of mathematics.

At Princeton University, Mr. PHILIP FRANKLIN, who received his doctorate there in June, has been appointed instructor of mathematics.

Dr. J. E. ROWE, recently of the Aberdeen Proving Ground, has been appointed head of the department of mathematics at College of William and Mary, Williamsburg, Va. Compare *1920*, 280.

In this case the correction $\Delta_4/6 = -.00000\ 00441 +$ and the value of $\sqrt[3]{62}$ is given to 9 places of decimal as above.—EDITOR.

At Lehigh University, Assistant Professors J. E. STOCKER and J. B. REYNOLDS have been promoted to associate professorships of mathematics and astronomy.

At Hobart College, Geneva, N. Y., where W. P. DURFEE, professor of mathematics, is also dean of the college, his son W. H. DURFEE has been appointed professor of freshman mathematics.

Dr. E. L. POST, of Columbia University, for the past year a fellow at Princeton University, has been appointed instructor of mathematics at Columbia University for the current academic year.

Mr. E. T. BROWNE, instructor of mathematics at the University of Virginia, but more recently a graduate student at the University of Chicago, has been appointed assistant professor of mathematics at Trinity College, Hartford, Conn.

DR. C. N. REYNOLDS, of Dartmouth College, (*1920*, 238), has been appointed assistant professor of mathematics at the University of West Virginia.

Assistant Professor E. S. ALLEN, of the University of West Virginia (*1919*, 419) has been appointed associate professor of mathematics in Iowa State College.

Associate Professor M. O. TRIPP, of the University of Maine, has been appointed head of the department of mathematics at Wittenberg College, Springfield, Ohio.

MR. E. L. MACKIE, instructor of mathematics at Harvard University during 1920–1921 (*1920*, 382), has been appointed assistant professor of mathematics at his Alma Mater, the University of North Carolina.

At Clark University, Dr. W. E. STORY has been made professor emeritus since the age limit for teachers there is seventy years. Dr. HENRY TABER has had to retire on account of ill health, and has also been made professor of mathematics, emeritus.

At Colgate University, Assistant Professor E. P. SISSON has retired from active teaching, Mr. TORWALD FREDERICKSEN (see *1921*, 235) has been given permanent appointment as instructor of mathematics, and Mr. H. A. DOBELL has also been made an instructor.

At Oberlin College, Mr. F. E. CARR has been promoted to an assistant professorship of mathematics, and Dr. C. H. YEATON, of Milwaukee College of Engineering, has been appointed assistant professor of mathematics.

At Knox College, Galesburg, Ill., Miss HELEN CALKINS, of Columbia University, has been appointed instructor of mathematics. Professor G. T. SELLEW, for twenty-two years head of the department of mathematics, has been granted a year's leave of absence for study and travel abroad. For a time he expects to study statistics with Karl Pearson in London.

Professor KATHARINE S. ARNOLD, who has been in the department of mathematics at Milwaukee-Downer College, Milwaukee, Wis., since 1912, has been appointed head of the department of mathematics at the Constantinople Woman's College, formerly known as the American College for Girls at Constantinople. She sailed in August to take up her new work.

Assistant Professor W. C. GRAUSTEIN, of Harvard University, and Assistant Professor MARY F. CURTIS, of Wellesley College, were married on June 10, 1921, and sailed for England on the following day. Professor Curtis had received leave of absence from Wellesley for 1921–1922.

One of the three exchange professors whom Harvard sends to France this year is one of our charter members, Professor A. E. KENNELLY, who goes under the auspices of a committee of American Universities interested in exchanging with France professors of engineering and applied science.

Dean T. F. HOLGATE, of Northwestern University, has been invited by the University of Nanking, China, to spend his sabbatic year at that institution, lecturing on mathematical subjects and assisting in the general organization of the university. He sailed for China on August 18.

At Harvard University, Dr. J. L. WALSH, Sheldon Fellow at Paris during the past year, has been appointed permanent instructor of mathematics (*1920*, 109, 440). The appointment as instructors of mathematics for 1921–1922 are as follows: Dr. C. E. HILLE as Benjamin Peirce instructor (see *1920*, 438); C. A. GARABEDIAN, R. E. LANGER and HARRY LEVY, who have been instructors during the past year (*1920*, 382); LINCOLN LA PAZ, L. L. SMITH (*1920*, 322) and R. M. FOSTER (*1921*, 106), of Harvard University; and H. W. BRINKMAN, of Stanford University (*1920*, 241). On August 9, 1921, LARNED LINN SMITH was drowned. He was a young mathematician of exceptional promise, aged twenty-two years. Dr. H. T. STETSON has been promoted to be an assistant professor of astronomy (*1921*, 285).

C. H. PEABODY, professor of naval architecture and marine engineering at the Massachusetts Institute of Technology since 1893, was made professor emeritus in June, 1920, after thirty-seven years of service in connection with the Institute. He was professor of mathematics and engineering at the Imperial Agricultural College in Sapporo, Japan, 1878–1880. His first work was *Thermodynamics of the Steam Engine*, 1889, which reappeared in several revised and enlarged editions, finally including the theory of turbines and internal combustion engines. His well known book on *Naval Architecture* was first published in 1904, and has gone through several editions.

Professor HORACE LAMB has been made professor of mathematics emeritus, at the University of Manchester, which, on May 7, conferred on him the degree D. Sc. He has been appointed to the honorary (Rayleigh) University lectureship at Cambridge.

WILLIAM ROBERT BROOKS, director of the Smith Observatory since 1888, and professor of astronomy at Hobart College since 1900, died at his home, Geneva, N. Y., on May 3, 1921. He was born in Kent, England, June 11, 1844, and came to this country in 1857. He founded the Red House Observatory, Phelps, N. Y., in 1874, and since that time discovered twenty-seven comets, more than any living astronomer has to his credit.

NATHANIEL FRENCH DAVIS, professor of pure mathematics, emeritus, in Brown University since 1915, died May 17, 1921. He was born at Lake Village (now Laconia), N. H., June 11, 1847, and graduated B. A. from Brown University in 1870. Returning to his Alma Mater in 1874 as instructor of mathematics, he was made assistant professor in 1879, associate professor in 1889, and professor of pure mathematics in 1890. We have recently referred (*1921*, 97) to the Fund of ten thousand dollars founded in honor of Professor Davis. His son, H. N. DAVIS, is professor of mechanical engineering at Harvard University (*1919*, 275).

Professor PAUL ARNOLD, head of the department of mathematics in University of Southern California since 1901, died February 24, 1921. He was born in Columbia, Missouri, December 16, 1870, but went to California with his father at an early age. Graduating from the University of Southern California, Ph.B., 1890, Ph.M., 1893, he taught in the department of mathematics 1890–1893. During 1894–1896 he was a graduate student in mathematics at Cornell University and held a fellowship from this University while studying at Berlin and Leipzig, 1896–1897. He remained abroad till 1899 studying with Lie at the University of Christiania.

Professor ALFRED MONROE KENYON, head of the department of mathematics at Purdue University, Indiana, since 1908, died while traveling on an interurban train after attending his mother's funeral, July 27, 1921. Born at Medina, O., December 10, 1869, he graduated from Hiram College and took graduate work at Western Reserve University, at the Case School of Applied Science, and at Harvard University (A.M., 1898). Appointed instructor of mathematics at Purdue University in 1898, he became an associate professor in 1901, and a professor in 1903. He was joint-author, with W. V. Lovitt, of *Mathematics for Collegiate Students of Agriculture and General Science*, New York, 1917; and with Louis Ingold, of (*a*) [*Plane and Spherical*] *Trigonometry*, New York, 1913; (*b*) *Elements of Plane Trigonometry*, New York, 1919.

The death is announced of M. FRANCOIS-JOSEPH PICAVET, secrétaire du Collège de France, directeur à l'Ecole Pratique des Hautes Etudes, chargé de cours à la Faculté des Lettres de l'Université de Paris, and chevalier de la légion d'honneur. M. Picavet died on May 19, 1921, at his residence in the Collège de France, at the age of seventy years, since he was born May 17, 1851. He is known in the history of mathematics for his great work on Gerbert,—*Gerbert, un pape philosophe, d'apres l'histoire et d'après la légende*, Paris, 1897. Few men of his time have done more to maintain high standards in the field of scholarly research.

ALBRECHT WILHELM ADOLF THAER, director of the Oberrealschule at Hamburg, died March 1, 1921. He was born January 13, 1855, and was granted his doctorate at Giessen in 1878. Apart from his dissertation, published in *Mathematische Annalen*, volume 14, his publications related to matters connected with secondary schools. He collaborated with others in the preparation of a series of elementary mathematical texts, 1911–1915, and of a report to the International Commission on Mathematical Instruction, 1911.

JOHANNES KARL THOMAE, ordinary professor of mathematics in Jena from 1879 to his retirement in 1914, died April 1, 1921. Born December 11, 1840, he received his doctorate at Göttingen in 1864. During the next fifty years he was a prolific writer of articles (especially in the *Berichte* of the Academy of Sciences at Leipzig), pamphlets, and books. Among his books were the following: (*a*) *Abriss einer Theorie der complexen Functionen und der Thetafunctionen einer Veränderlichen*, Halle, 1870; third edition, 1890; (*b*) *Elementare Theorie der analytischen Functionen einer complexen Veränderlichen*, Halle, 1880; second edition, 1898; (*c*) *Die Kegelschnitte in rein projectivischer Behandlung*, Halle, 1894; (*d*) *Grundriss einer analytischen Geometrie der Ebene*, Leipzig, 1906.

Mr. ALFRED DAVIS, of the Soldan High School, St. Louis, is president of the Mathematics Club of St. Louis (see *Mathematics Teacher*, 1921, page 156).

Professor W. C. EELLS, of Whitman College, Wash., is chairman of the mathematics section of the Inland Empire Teachers Association.

On June 10, Professor A. G. WEBSTER, of Clark University, delivered before the Royal Institution of Great Britain a lecture on "Researches on sound."

The inauguration of Dr. E. F. NICHOLS as president of Massachusetts Institute of Technology occurred June 8; his address was published in *Science*, June 10.

Dr. J. S. TAYLOR, of Massachusetts Institute of Technology, has been granted a year's leave of absence which he is spending at the University of Louvain as a Belgian-American fellow.

Professor R. C. ARCHIBALD, of Brown University, was on June 20, 1921, elected an honorary member of the Harvard Chapter of Phi Beta Kappa. Brown University has granted him leave of absence for the second half of the academic year, 1921–1922. He expects to spend it in visiting mathematicians at universities of Italy, France, Belgium, Holland, Scandinavia, and Great Britain.

Miss ANNIE J. CANNON, who has been connected with the Harvard Observatory since 1897, and curator of the astronomical photographs there since 1911, has received from Groningen University, Holland, an honorary doctor's degree in mathematics and astronomy, in acknowledgment of her work in the study of stellar spectra which will fill nine quarto volumes of the *Annals of the Harvard Observatory;* three of these are now published.

The Committee on Algebraic Numbers, formed in 1920 by the Division of Physical Sciences of the National Research Council, is composed of Professors H. H. MITCHELL, H. S. VANDIVER, G. E. WAHLIN, and L. E. DICKSON, chairman. At its meeting in Philadelphia, April 20–22, 1921, final plans were formulated for the preparation of a report on the literature of algebraic numbers subsequent to Hilbert's report, "Die Theorie der algebraischen Zahlkörper," in *Jahresbericht der deutschen Mathematiker-Vereinigung*, 1897.

At the inauguration of L. T. COFFMAN (joint author, with J. C. Brown, of *How to Teach Arithmetic*, Chicago, 1914; and, with W. A. Jessup, of *The Supervision of Arithmetic*, New York, 1916) as president of the University of Minnesota,

Professor W. H. BUSSEY, of the University of Minnesota, was the delegate from the American Mathematical Society, and Professor H. E. SLAUGHT from the Mathematical Association of America.

Among the delegates at the inauguration on June 22, 1921, of Dr. J. R. Angell, president of Yale, were the following: Dr. E. G. BILL, Dartmouth College; Acting Dean JAMES HARKNESS, McGill University; President W. A. GRANVILLE, Pennsylvania College; Professor A. S. GALE, University of Rochester; Professor JOSEPH BOWDEN, Adelphi College; President E. O. LOVETT, Rice Institute; Professor R. G. D. RICHARDSON, American Mathematical Society; Professor E. W. BROWN, American Philosophical Society.

At the meeting of the American Academy of Arts and Sciences in May, 1921, the following fellows were elected in Section I—Mathematics and Astronomy: C. G. ABBOT, astrophysicist, Smithsonian Institution; FLORIAN CAJORI, historian of mathematics, University of California; O. D. KELLOGG, mathematican, Harvard University; H. N. RUSSELL, astronomer, Princeton University; FRANK SCHLESINGER, astronomer, Yale University; JOEL STEBBINS, astronomer, University of Illinois. G. H. HARDY, mathematician, of Oxford University, was elected a foreign honorary fellow. Among officers of the Academy elected for the ensuing year were H. W. TYLER, corresponding secretary, A. G. WEBSTER, librarian, and E. B. WILSON, a member of the "committee on meetings."

At the meeting on April 11 of the Academy of Sciences of the Institute of France (see *1920*, 384), EMILE BOREL, professor of the calculus of probabilities and of mathematical physics at the University of Paris, was elected to succeed HUMBERT (see *1921*, 237) as one of the six members in the section of geometry. There are in the Academy, apart from its two permanent secretaries one in each of the departments, physics and mathematics, 66 regular members, six in each of the eleven sections: geometry, mechanics, astronomy, geography and navigation, general physics, chemistry, mineralogy, botany, agriculture, anatomy and zoology, and medicine and surgery.

At the meeting of the Southwestern Michigan Association of Teachers of Mathematics and Science held in Kalamazoo on April 22, 1921, H. E. SLAUGHT spoke on behalf of the National Committee on Mathematical Requirements concerning the history and work of this committee.

At a meeting of the Twin Cities Mathematical Club held at the University of Minnesota on May 14, 1921, H. E. SLAUGHT addressed the high school teachers of St. Paul and Minneapolis on "Certain marvelous magnifications of mathematics." The club of some sixty members was organized two years ago by W. D. REEVE, head of the department of mathematics in the University of Minnesota High School.

At the fourteenth regular meeting of the Association of Mathematics Teachers of New Jersey, at Princeton, N. J., May 21, 1921, the following papers were read: "A high school test in arithmetic" by A. W. BELCHER; "Some geometry propositions proven by elementary mechanics" (presidential address) by C. R. MAC-

INNES; "Recent developments in connection with the work of the National Committee" by H. E. WEBB; "A geometric introduction to the theory of relativity" by L. P. EISENHART.

The twenty-fourth regular meeting of the Rochester section of the Association of Teachers of Mathematics in the Middle States and Maryland was held at the University of Rochester on May 7, and the following papers were read: "The place of the formula in elementary algebra" by ELIZABETH M. LOETZER; "Experiences based on the report of the National Committee on Mathematical Requirements" by WILLIAM BETZ; "Discussion of Einstein's theory of relativity" by A. S. GALE.

In our report (*1921*, 239) of the meeting of the Mathematics Section of the New York State Teachers Association at Rochester, November 23, 1920, record of G. R. MIRICK's paper, "The proper correlation of intermediate algebra, trigonometry, and solid geometry, in the senior high school," was unintentionally omitted. A similar omission was made (*1921*, 240) in the case of M. M. S. MORIARTY'S paper, "Some geometric notions," read at the meeting of the Connecticut Valley Section of the Association of Teachers of Mathematics in New England, November 6, 1920.

At the meetings of the American Mathematical Society at Wellesley College, September 8–9, 36 papers were presented by the following mathematicians: J. W. ALEXANDER (2), G. D. BIRKHOFF (2), B. H. CAMP, W. L. CRUM (2), E. L. DODD, JOHN EIESLAND, L. P. EISENHART, C. A. FISCHER, R. E. GILMAN, O. E. GLENN, T. H. GRONWALL, C. F. GUMMER, EUGENIE C. HAUSLE, E. V. HUNTINGTON, EDWARD KASNER, J. R. KLINE (2), G. A. MILLER, C. N. MOORE, R. L. MOORE, H. M. MORSE, J. R. MUSSELMAN, M. B. PORTER, J. E. ROWE, I. J. SCHWATT (3), J. L. SYNGE, J. L. WALSH, A. G. WEBSTER (2), N. WIENER, S. D. ZELDIN. Details concerning the papers may be found in *Bulletin of the American Mathematical Society*, January, 1922.

At Göttingen on January 6, 1921, was organized the Reichsverband deutscher mathematischer Gesellschaften und Vereine (National Association of German Mathematical Societies and Clubs). The members of the Association are: the Deutsche Mathematiker-Vereinigung, the Deutscher Verein zur Förderung des mathematischen und naturwissenschaftlichen Unterricht, the Mathematisch-Naturwissenschaftlicher Verein in Württemberg and the mathematical societies in Berlin, Vienna, Hamburg, Göttingen and Aix-la-Chapelle. It is desired to unite in this Association all mathematical societies and clubs of German speaking countries, whether their aim is primarily educational or scientific. In this way it is expected that the ideals of all will be more effectively promoted, that the position of mathematics in practical life will in every direction be represented, and that especially, the interests of mathematical instruction of every grade will be guarded.

RELIGIO MATHEMATICI.

PRESIDENTIAL ADDRESS DELIVERED BEFORE THE MATHEMATICAL ASSOCIATION OF AMERICA, SEPTEMBER 7, 1921.

By DAVID EUGENE SMITH, Columbia University.

Apologia. Well aware that I, today, break all precedents in the nature of a presidential address, it is proper that I offer due apology and confess my unconventionality. It is proper, too, that I should frankly say that I am aware that my message is wanting in the rigor of demonstration to which our science accustoms us; that it involves no mathematics beyond the merest elements; that it will be listened to with regret by some and with disapproval by others; and that it is likely to carry conviction to only a limited number.

Why, then, should I choose such a topic? Why should I force my friends to apologize for my thoughts, and why should I place others in a favorable position to condemn my action? The answer is a simple one:—I believe it to be a duty laid upon us, that one of our number should, on some such occasion as this, say what I shall say. I regret that another,—one with more faith, one better endowed with power of expression, one whose words would carry greater weight than any I can utter,—might not have had the opportunity that is mine to deliver the message. And finally, by way of apology, I say with perfect candor that I speak not at all to the audience here present,—for what I shall say will convey nothing that is new to any one of you; I speak rather to those who are not present, those who are not of our special guild, and those to whom the teaching of mathematics may possibly, by this means, appear in a somewhat different, and I hope in a somewhat more favorable light.

The General Message. It is nearly three centuries ago that Sir Thomas Browne wrote, for private circulation among a few of his intimate friends, his *Religio Medici*. He did not call it *Religio Medicorum*, for he could not assume to speak for others. Indeed, as he confessed, the men who practiced the healing art in his time were generally, like loud-mouthed boys in their teens, boastful of their atheism; but for himself, a humble practitioner in an ancient town of England, he could testify; and he, the giver of balm for the body, could unobtrusively pass the message to others, as a balm for the spirit.

And so, today, I speak not of the *Religio Mathematicorum*, for I have not the authority; I feel, indeed, that *Religio Mathematici* is a misnomer; but I also feel that it is proper to speak of the relation of mathematics to a religious attitude of mind, and I know of no better title for my purpose than the one I have chosen.

Do not, however, feel that the message seeks to change the faith or lack of faith of any man; do not feel that it contains a plea for any creed or for any sect; do not feel that it sets forth anything that is new; but rather feel that it represents the mere commonplace knowledge that most of us have, and the mere basis of belief that mathematics may possibly foster. It goes no farther, it seeks only a